2-ma‘ruza. Mavzu: Uch o’lchovli integral

Reja:
1. Massa haqidagi masala.
2. Uch o’lchovli integral va uning xossalari.
3. Uch o’lchovli integralni hisoblash.
4. Uch o’lchovli integralda o’zgaruvchilarni almashtirish.
5. Uch o’lchovli integralning tadbiqlari.

2.1. Massa haqidagi masala

$0xyz$ fazoda V moddiy jism berilgan bo‘lsin. Uning $P(x,y,z)$ nuqtasini o‘z ichiga oluvchi ΔV kichik qismini qaraymiz. Shu kichik bo‘lakning Δm massasini uning ΔV hajmiga nisbati $\dfrac{\Delta m}{\Delta V}$ ΔV jismning **o‘rtachi zichligi** deb ataladi.

$\dfrac{\Delta m}{\Delta V}$ nisbatning ΔV kichik jism $P(x,y,z)$ nuqtaga tortilgandagi limiti γ V jismning $P(x,y,z)$ nuqtadagi **zichligi** deb ataladi. U $P(x,y,z)$ nuqtaning vaziyaiga bog‘liq, ya’ni uning koordinatalarini funksiyasi bo‘ladi: $\gamma = \gamma(x,y,z)$. V jismning istalgan $P(x,y,z)$ nuqtasida $\gamma = \gamma(x,y,z)$ chiziqli zichlik uzluksiz funksiya sifatida berilganda uning m massasini topamiz. Agar V jism bir jinsli bo’lganda uning barcha nuqtalarida chiziqli zichlik bir xil γ_0 ga teng bo‘lib uning massasi

$$m = \gamma_0 V \qquad (2.1)$$

bo‘ladi, bunda V - jismning hajmi.

Umumiy holda γ chiziqli zichlik nuqtaning holatiga qarab o‘zgarganda (2.1) formula massani aniqlashga yaroqli emas. Shuning uchun V jismni shunday n ta $\Delta V_1, \Delta V_2, ..., \Delta V_n$ kichik jismlarga ajratamizki ularning hajmlarining yig‘indisi berilgan jismning hajmi V ga teng bo‘lsin, ya’ni $V = \sum_{k=1}^{n} \Delta V_k$. Har bir ΔV_k kichik jismda bittadan $P_k(x_k, y_k, z_k)$ nuqtani tanlaymiz. Agar ΔV_k ni yetarlicha kichik jismda uning chiziqli zichligi P_k nuqtadagi chiziqli zichlik $\gamma_k = \gamma(x_k, y_k, z_k)$ dan juda kam farq qiladi. Har bir ΔV_k jismning barcha nuqtalarida chiziqli zichlik o‘zgarmas va u P_k nuqtadagi chiziqli zichlikka teng deb hisoblab uning Δm_k massasini hisoblaymiz

$$\Delta m_k \approx \gamma_k \Delta V_k = \gamma(x_k, y_k, z_k)\Delta V_k \quad (k = \overline{1,n}).$$

Butun jismning m massasi $\sum_{k=1}^{n} \Delta m_k$ ga teng bo‘lgani uchun uni hisoblash uchun

$$m = \sum_{k=1}^{n} \Delta m_k \approx \sum_{k=1}^{n} \gamma(x_k, y_k, z_k)\Delta V_k$$

taqribiy formulaga ega bo‘lamiz.

ΔV_k kichik jismlarning har biri nuqtaga tortilganda bu yig‘indining limitini berilgan jismning massasi sifatida qabul qilamiz:

$$m = \lim_{\max diam\Delta V_k \to 0} \sum_{k=1}^{n} \gamma(x_k, y_k, z_k)\Delta V_k \,.$$

Jismning massasini topish masalasi bizni ma’lum yig‘indining limitini topishga olib keldi. Ko‘pgina masalalarning yechimi bu kabi yig‘indining limitini topishga keltiriladi. Shuning uchun bu kabi masalalarni fizik ma’nosidan qat’iy nazar yig‘indining limitini umumiy holda o‘rganishga kirishamiz.

2.2.Uch o’lchovli integral va uning xossalari

$0xyz$ fazoda V hajmga ega bo‘lgan jism berilgan bo‘lsin. Bu sohaning har bir $P(x, y, z)$ nuqtasida $u = f(x, y, z)$ funksiya aniqlangan bo‘lsin. Quyidagi amallarni bajaramiz.

1.Berilgan jismni hajmlarining yig‘indisi berilgan jismning hajmi V ga teng n ta ixtiyoriy kichik $\Delta V_1, \Delta V_2, ..., \Delta V_n$ jismlarga ajratamiz.

2. Har bir kichik ΔV_k jismda ixtiyoriy $P_k(x_k, y_k, z_k)$ nuqtani tanlaymiz. $u = f(x, y, z)$ funksiyaning P_k nuqtadagi qiymatini ΔV_k kichik jismning hajmi ΔV_k ga ko‘paytirib

$$f(P_k)\Delta V_k = f(x_k, y_k, z_k)\Delta V_k$$

ifodani tuzamiz.

3. Shu kabi ko‘paytmalarni yig‘indisi

$$\sum_{k=1}^{n} f(P_k)\Delta V_k = \sum_{k=1}^{n} f(x_k, y_k, z_k)\Delta V_k \qquad (2.2)$$

ni tuzamiz. Bu yig‘indi V sohada $f(x, y, z)$ funksiya uchun **integral yig‘indi** deyiladi.

1-ta’rif. (2.2) integral yig‘indi $\Delta V_1, \Delta V_2, ..., \Delta V_n$ mayda jismlarning diametrlarining eng kattasi 0 ga intilganda V sohani $\Delta V_1, \Delta V_2, ..., \Delta V_n$ qismlarga bo‘linish usuli hamda bu bo‘laklarda $P_1, P_2, ..., P_n$ nuqtalarning tanlanishiga bog‘liq bo‘lmagan aniq limitga ega bo‘lsa, bu limit $f(x, y, z)$ funksiyadan V soha bo‘yicha olingan uch o‘lchovli integral deyiladi va

$$\iiint_V f(P)dV, \text{ yoki } \iiint_V f(x, y, z)dxdydz, \text{ yoki } \iiint_V f(x, y, z)dV$$

ko‘rinishda belgilanadi.

Demak ta’rifga binoan

$$\lim_{\max diam\Delta V_k \to 0} \sum_{k=1}^{n} f(P_k)\Delta V_k = \iiint_V f(x, y, z)dV \,.$$

Jismning massasi haqidagi masalaga qaytib V jismning m massasi $\gamma = \gamma(x, y, z)$ chiziqli zichlikdan V soha bo‘yicha olingan uch o‘lchovli integralga teng ekanligiga iqror bo‘lamiz, ya’ni

$$m = \iiint_V \gamma(x, y, z)dxdydz.$$

2-ta’rif. Chekli limit $\lim\limits_{\max diam\Delta V_k \to 0} \sum\limits_{k=1}^{n} f(P_k)\Delta V_k$ mavjud bo‘lsa, u holda $f(x, y, z)$ funksiya V sohada **integrallanuvchi** deb ataladi.

2.1-teorema. V hajmga ega V sohada uzluksiz $u=f(x,y,z)$ funksiya shu sohada integrallanuvchidir.

Uch o’lchovli integral ikki o’lchovli integralning umumlashgan holi bo’lgani uchun u ikki o’lchovli integral ega bo’lgan barcha *xossalarga* ega.

1-xossa. O‘zgarmas ko‘paytuvchini uch o‘lchovli integral belgisidan tashqariga chiqarish mumkin, ya’ni

$$\iiint_V kf(x,y,z)dxdydz = k\iiint_V f(x,y,z)dxdydz,$$

bunda k -o`zgarmas son.

2-xossa. Bir nechta integrallanuvchi funksiyalarning algebraik yig‘indisidan olingan uch o‘lchovli integral qo‘shiluvchilardan olingan uch o‘lchovli integrallarning algebraik yig‘indisiga teng:

$$\iiint_V [f(x,y,z)\pm\varphi(x,y,z)]dxdydz = \iiint_V f(x,y,z)dxdydz \pm \iiint_V \varphi(x,y,z)dxdydz.$$

3-xossa. Agar V integrallash soha umumiy ichki nuqtalarga ega bo‘lmagan k ta $V_1, V_2, ..., V_k$ qismlarga ajratilsa, u holda integrallanuvchi funksiyadan butun V soha bo‘yicha olingan uch o‘lchovli integral uning har qaysi qismlari bo‘yicha olingan uch o‘lchovli integrallarning yig‘indisiga teng:

$$\iiint_V f(x,y,z)dxdydz = \iiint_{V_1} f(x,y,z)dxdydz + \iiint_{V_2} f(x,y,z)dxdydz + ... + \iiint_{V_n} f(x,y,z)dxdydz.$$

4-xossa. Agar V sohada integrallanuvchi $f(x,y,z)$ funksiya shu sohada $f(x,y,z)\geq 0$ bo‘lsa, u holda $\iiint_V f(x,y,z)dxdydz \geq 0$; V sohada $f(x,y,z)\leq 0$ bo‘lsa, u holda $\iiint_V f(x,y,z)dxdydz \leq 0$ bo‘ladi.

5-xossa. Agar V sohada $f(x,y,z)$, $\varphi(x,y,z)$ funksiyalar integrallanuvchi va $f(x,y,z)\geq\varphi(x,y,z)$ bo‘lsa, u holda

$$\iiint_V f(x,y,z)dxdydz \geq \iiint_V \varphi(x,y,z)dxdydz$$

bo‘ladi.

6-xossa (o‘rta qiymat haqida). Agar $f(x,y,z)$ funksiya yopiq chegaralangan V sohada uzluksiz bo‘lsa, u holda bu sohada shunday $P_0(x_0,y_0,z_0)$ nuqta mavjud bo‘lib

$$\iiint_V f(x,y,z)dxdydz = f(x_0,y_0,z_0)\cdot V$$

tenglik bajariladi, bu yerdagi V-sohaning hajmi. Funksiyaning $f(x_0,y_0,z_0)=\frac{1}{V}\iiint_V f(x,y,z)dxdydz$ qiymati uning V sohadagi **o‘rta qiymati** deyiladi.

7-xossa (integralning chegaralanganligi haqida). Agar $f(x,y,z)$ funksiya yopiq chegaralangan V sohada uzluksiz hamda M va m uning shu sohadagi eng katta va eng kichik qiymatlari bo‘lsa, u holda

$$m\cdot V \leq \iiint_V f(x,y,z)dxdydz \leq M\cdot V$$

tengsizlik o‘rinli, bu yerdagi V-sohaning hajmi. Shuni aytish joizki V sohada integral ostidagi $f(x,y,z)\equiv 1$ bo‘lganda uch o‘lchovli integral son qiymat bo‘yicha sohaning hajmiga teng, ya’ni

$$\iiint_V dxdydz = V .$$

Bu qaralayotgan holda istalgan integral yig‘indi $\sum_{k=1}^{n} 1\cdot\Delta V_k = V$ ko‘rinishga ega ekanligidan kelib chiqadi.

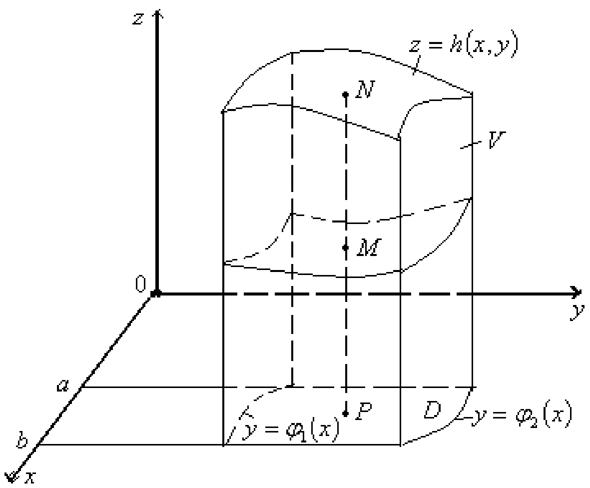
2.3. Uch o’lchovli integralni hisoblash

Uch o‘lchovli integralni hisolash uchta aniq integralni ketma-ket hisoblashga keltiriladi. Faraz qilaylik integrallash sohasi V quyidan uzluksiz $z=g(x,y)$ sirt bilan, yuqoridan uzluksiz $z=h(x,y)$ sirt bilan chegaralangan jism bo‘lsin. Bu jism $0xy$ tekisligiga $y=\varphi_1(x), y=\varphi_2(x), [\varphi_1(x)\le\varphi_2(x)]$ uzluksiz egri chiziqlar bilan, hamda $x=a, x=b\,(a<b)$ vertikal to‘g‘ri chiziqlar bilan chegaralangan D sohaga proyeksiyalansin. D sohaning $P(x,y,0)$ nuqtasi orqali $0z$ o‘qqa parallel to‘g‘ri chiziq o‘tkazamiz. Bu to‘g‘ri chiziq $z=g(x,y)$ sirt bilan M nuqtada, $z=h(x,y)$ sirt bilan N nuqtada uchrashadi.

$f(x,y,z)$ funksiya V sohada uzluksiz bo‘lganda $\iiint_V f(x,y,z)dxdydz$ uch o‘lchovli integralni hisoblash uchun

$$\iiint_V f(x,y,z)dxdydz = \iint_D \left\{ \int_{g(x,y)}^{h(x,y)} f(x,y,z)dz \right\} dxdy \qquad (2.3)$$

formula o‘rinliligini isbotlash mumkin. Bu formulaga shunday izoh beriladi. $\iiint_V f(x,y,z)dxdydz$ uch o‘lchovli integralni hisoblash uchun avval $\int_{g(x,y)}^{h(x,y)} f(x,y,z)dz$ aniq integralni x va y ni o‘zgarmas sanab hisoblanadi. Integralning quyi chegarasi M nuqtaning applikatasi $g(x,y)$ dan va yuqori chegarasi N nuqtaning applikatasi $h(x,y)$ dan iborat. Integralni hisoblash natijasida x va y ning qandaydir funksiya hosil bo‘ladi. Bu funksiyaning V jismning $0xy$ tekislikdagi proyeksiyasi D bo‘yicha integrallab uch o‘lchovli integralning qiymati topiladi. $\int_{g(x,y)}^{h(x,y)} f(x,y,z)dz$ funksiyadan D soha bo‘yicha ikki o‘lchovli integralni hisoblab



1-chizma.

$$\iint_D \left\{ \int_{g(x,y)}^{h(x,y)} f(x,y,z)dz \right\} dxdy = \int_a^b dx \int_{\varphi_1(x)}^{\varphi_2(x)} \left\{ \int_{g(x,y)}^{h(x,y)} f(x,y,z)dz \right\} dy$$

tenglikka ega bo‘lamiz.

Shunday qilib,

$$\iiint_V f(x,y,z)dxdydz = \int_a^b dx \int_{\varphi_1(x)}^{\varphi_2(x)} \left\{ \int_{g(x,y)}^{h(x,y)} f(x,y,z)dz \right\} dy \quad (2.4)$$

uch o‘lchovli integralni hisoblash formulasiga ega bo‘ldik.

(2.4) tenglikning o‘ng tomonidagi ifoda **uch karrali integral** deb ataladi.

Agar V soha qaralganga nisbatan ancha murakkab bo‘lsa, u holda V sohani chekli sondagi qaralgan shakldagi $V_1, V_2, ..., V_k$ sohalarga ajratilib ularning har biriga (2.4) formula qo‘llaniladi. V soha bo‘yicha uch o‘lchovli integral $V_1, V_2, ..., V_k$ sohalar bo‘yicha uch o‘lcho vli integrallarning yig‘indisiga teng bo‘ladi.

2.4. Uch o’lchovli integralda o’zgaruvchilarni almashtirish

Ushbu

$$\begin{cases} x = \varphi(u,t,w), \\ y = \psi(u,t,w), \\ z = g(u,t,w). \end{cases}$$

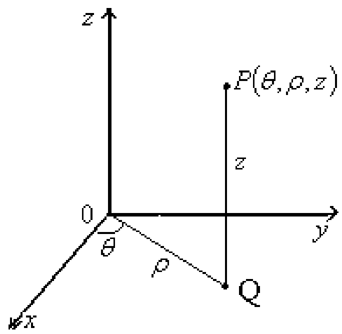
funksiyalar x, y, z dekart koordinatalarida berilgan V sohani u, t, w koordinatalardagi V' sohaga o’zaro bir qiymatli **akslantiradi** deb faraz qilamiz. U holda

$$\iiint_V f(x,y,z)dxdydz = \iiint_{V'} f(\varphi(u,t,w), \psi(u,t,w), g(u,t,w)|I|dudtdw \quad (2.5)$$

o’zgaruvchilarni almashtirish formulasi o’rinlidir.

Yakobian deb ataluvchi I quyidagicha aniqlanadi:

$$I = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial t} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial t} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial t} & \frac{\partial z}{\partial w} \end{vmatrix} \neq 0 .$$



2-chizma.

Endi fazodagi nuqtaning dekart koordinatalaridan silindrik koordinatalariga o‘tganda o‘zgaruvchilarni almashtirish formulalari bilan tanishamiz.

$0xyz$ koordinatalar sistemasidagi P nuqtani qaraymiz. $Q-P$ nuqtaning $0xy$ tekislikdagi proyeksiyasi bo‘lsin. P nuqtaning fazodagi vaziyatini Q nuqtaning $0xy$ tekislikdagi qutb koordinatalari θ, ρ hamda P nuqtaning z applikatasi bilan aniqlanadi. θ, ρ, z sonlar P nuqtaning silindrik koordinatalari deb ataladi (2-chizma). Nuqtaning silindrik koordinatalari uning x, y, z dekart koordinatalari bilan

$$x = \rho\cos\theta,\ y = \rho\sin\theta,\ z = z$$

munosabat bilan bog‘langan.

Endi silindrik koordinatalarga o‘tilganda $|I|$ ni topamiz $(u = \rho,\ t = \theta,\ w = z)$.

$$\frac{\partial x}{\partial \theta} = (\rho\cos\theta)_\theta' = -\rho\sin\theta,\ \frac{\partial x}{\partial \rho} = (\rho\cos\theta)_\rho' = \cos\theta,\ \frac{\partial x}{\partial z} = (\rho\cos\theta)_z' = 0,$$

$$\frac{\partial y}{\partial \theta} = (\rho\sin\theta)_\theta' = \rho\cos\theta,\ \frac{\partial y}{\partial \rho} = (\rho\sin\theta)_\rho' = \sin\theta,\ \frac{\partial y}{\partial z} = (\rho\sin\theta)_z' = 0,$$

$$\frac{\partial z}{\partial \theta} = (z)_\theta' = 0,\ \frac{\partial z}{\partial \rho} = (z)_\rho' = 0,\ \frac{\partial z}{\partial z} = (z)_z' = 1.$$

$$I = \begin{vmatrix} \cos\theta & -\rho\sin\theta & 0 \\ \sin\theta & \rho\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \cos\theta & -\rho\sin\theta \\ \sin\theta & \rho\cos\theta \end{vmatrix} = \rho\cos^2\theta + \rho\sin^2\theta = \rho.$$

Shunday qilib

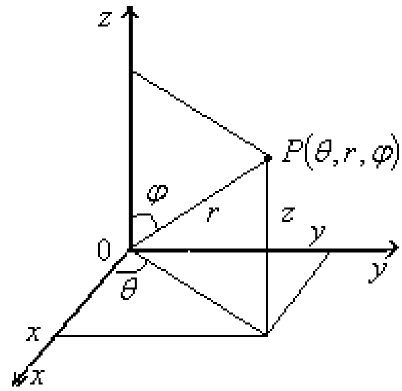
$$\iiint_V f(x,y,z)dxdydz = \iiint_{V'} f(\rho\cos\theta, \rho\sin\theta, z)\,\rho d\rho d\theta dz \qquad (2.6)$$

dekart koordinatalaridan silindrik koordinatalarga o‘tish formulasiga ega bo‘lamiz.

Fazoda nuqtaning x, y, z dekart koordinatalari, $\theta,\ \rho,\ z$ silindrik koordinatalari bilan bir qatorda uning sferik koordinatalari deb ataluvchi θ, r, φ koorinatalari ham keng qo‘llaniladi (3-chizma). Nuqtaning dekart va sferik koordinatalari orasida

$x = r\sin\varphi\cos\theta,\ y = r\sin\varphi\sin\theta,\ z = r\cos\varphi$

munosabat mavjud. Fazoning ixtiyoriy nuqtasi uchun: $0 \le r < +\infty, 0 \le \varphi \le \pi,\ 0 \le \theta \le 2\pi$.



3-chizma.

Dekart koordinatalaridan sferik koordinatalarga o‘tganda yakobian $(r = u,\ \varphi = t,\ \theta = w)$

$$I = \begin{vmatrix} \sin\varphi\cos\theta & r\cos\varphi\cos\theta & -r\sin\varphi\sin\theta \\ \sin\varphi\sin\theta & r\cos\varphi\sin\theta & r\sin\varphi\cos\theta \\ \cos\varphi & -r\sin\varphi & 0 \end{vmatrix} = r^2\sin\varphi$$

bo’ladi.

2.5.Uch o‘lchovli integralning tadbiqlari

Jismning statik momenti. Ma’lumki, m massali moddiy nuqtaning $0xy$ tekislikka nisbatan statik momenti S_{xy} deb nuqtaning massasini uning applikatasiga ko‘paytmasiga aytiladi: $S_{xy} = m \cdot z$. $0yz$ va $0xz$ tekisliklarga nisbatan statik momentlar S_{yz} va S_{xz} ham shunga o‘xshash aniqlanadi: $S_{yz} = mx$, $S_{xz} = my$. Agar bir nechta moddiy nuqtalardan tashkil topgan sistema berilgan bo‘lsa, u holda bu sistemaning statik momenti shu sistemani tashkil etuvchi nuqtalarning statik momentlarining yig‘indisiga teng bo‘ladi.

Faraz qilaylik fazoda V jism berilgan bo‘lib uning istalgan nuqtasida zichlik shu nuqta koordinatalarining uzluksiz funksiyasi sifatida ma’lum bo‘lsin, $\gamma = \gamma(x, y, z)$.

Shu jismning S_{xy} statik momentini topamiz. V jismni n ta ixtiyoriy kichik ΔV_k $\left(k = \overline{1, n}\right)$ jismlarga ajratib ularning har birida bittadan ixtiyoriy $P_k(x_k, y_k, z_k)$ nuqtani tanlaymiz. ΔV_k kichik jismning barcha nuqtalarida zichlik o‘zgarmas, hamda tanlangan P_k nuqtadagi zichlikka teng deb hisoblab ΔV_k jismning Δm_k massasi uchun

$$\Delta m_k \approx \gamma(x_k, y_k, z_k)\Delta V_k$$

taqribiy ifodaga ega bo‘lamiz. Har bir ΔV_k kichik jismni massasi Δm_k bo‘lgan $P_k(x_k, y_k, z_k)$ nuqtaga almashtiramiz. Bu nuqtaning $0xy$ tekislikka nisbatan statik momenti ΔV_k jismning $0xy$ tekislikka nisbatan statik momenti-ΔS_{xy}^k ning taqribiy qiymatini ifodalaydi:

$$\Delta S_{xy}^k \approx z_k \Delta m_k \approx z_k \gamma(x_k, y_k, z_k)\Delta V_k$$

V jismning $0xy$ tekislikka nisbatan statik momenti S_{xy} uchun

$$S_{xy} \approx \sum_{k=1}^{n} \Delta S_{xy}^k \approx \sum_{k=1}^{n} z_k \gamma(x_k, y_k, z_k)\Delta V_k$$

taqribiy tenglikka ega bo‘lamiz. Bunda $\max diam \Delta V_k \to 0$ da (ΔV_k nuqtaga tortilganda) limitga o‘tib

$$S_{xy} = \iiint_V z\gamma(x, y, z)dV \qquad (2.7)$$

statik momentning aniq qiymatini topish formulasini hosil qilamiz.

Shunga o‘xshash V jismning $0yz$ va $0xz$ tekisliklarga nisbatan statik momentlari uchun

$$S_{yz} = \iiint_V x\gamma(x, y, z)dV, \quad S_{xz} = \iiint_V y\gamma(x, y, z)dV \qquad (2.8)$$

formulalarga ega bo‘lamiz.

Jismning og‘irlik markazi. V jismning og‘irlik markazi $C(x_C, y_C, z_C)$ ning koordinatalari uchun

$$x_C = \frac{S_{yz}}{m}, \quad y_C = \frac{S_{xz}}{m}, \quad z_C = \frac{S_{xy}}{m} \qquad (2.9)$$

formulalar o‘rinli, bunda $m - V$ jismning massasi. V jismning massasi

$$m = \iiint_V \gamma(x, y, z)dV$$

kabi topilar edi. (2.7), (2.8) va (2.9) formulalarga asosan

$$x_C = \frac{\iiint_V x\gamma(x, y, z)dV}{\iiint_V \gamma(x, y, z)dV}, \quad y_C = \frac{\iiint_V y\gamma(x, y, z)dV}{\iiint_V \gamma(x, y, z)dV}, \quad z_C = \frac{\iiint_V z\gamma(x, y, z)dV}{\iiint_V \gamma(x, y, z)dV} \qquad (2.10)$$

V jismning og‘irlik markazini koordinatalarini topish formulasiga ega bo‘lamiz.

Bir jinsli jism uchun $\gamma = const$ bo‘lgani uchun (2.10) formulalar

$$x_C = \frac{\iiint\limits_V x dV}{\iiint\limits_V dV}, \quad y_C = \frac{\iiint\limits_V y dV}{\iiint\limits_V dV}, \quad z_C = \frac{\iiint\limits_V z dV}{\iiint\limits_V dV} \qquad (2.11)$$

ko‘rinishga ega bo‘ladi. $\iiint\limits_V dV = V$ ekanini hisobga olsak

$$x_C = \frac{1}{V}\iiint\limits_V x dV, \quad y_C = \frac{1}{V}\iiint\limits_V y dV, \quad z_C = \frac{1}{V}\iiint\limits_V z dV \qquad (2.12)$$

kelib chiqadi, bunda V jismning hajmi.

Jismning inersiya momenti. m massali $M(x, y, z)$ moddiy nuqtaning $0x$, $0y$, $0z$ koordinata o‘qlariga nisbatan inersiya momenti mos ravishda

$$I_{xx} = \left(y^2 + z^2\right)m, \quad I_{yy} = \left(x^2 + z^2\right)m, \quad I_{zz} = \left(x^2 + y^2\right)m$$

formula bilan ifodalanadi.

Jismning inersiya momenti mos uch o‘lchovli integrallar bilan ifodalanadi. Masalan, V jismning $0z$ o‘qqa nisbatan inersiya momenti

$$I_{zz} = \iiint\limits_V \left(\mathrm{x}^2 + y^2\right)\gamma(x, y, z)dV, \qquad (2.13)$$

bunda $\gamma(x, y, z)$ moddaning zichligi.

Shunga o‘xshash V jismning $0x$ va $0y$ o‘qlarga nisbatan inersiya momentlarini topish uchun

$$I_{xx} = \iiint\limits_V \left(\mathrm{y}^2 + z^2\right)\gamma(x, y, z)dV, \quad I_{yy} = \iiint\limits_V \left(\mathrm{x}^2 + z^2\right)\gamma(x, y, z)dV$$

formulalarga ega bo‘lamiz.