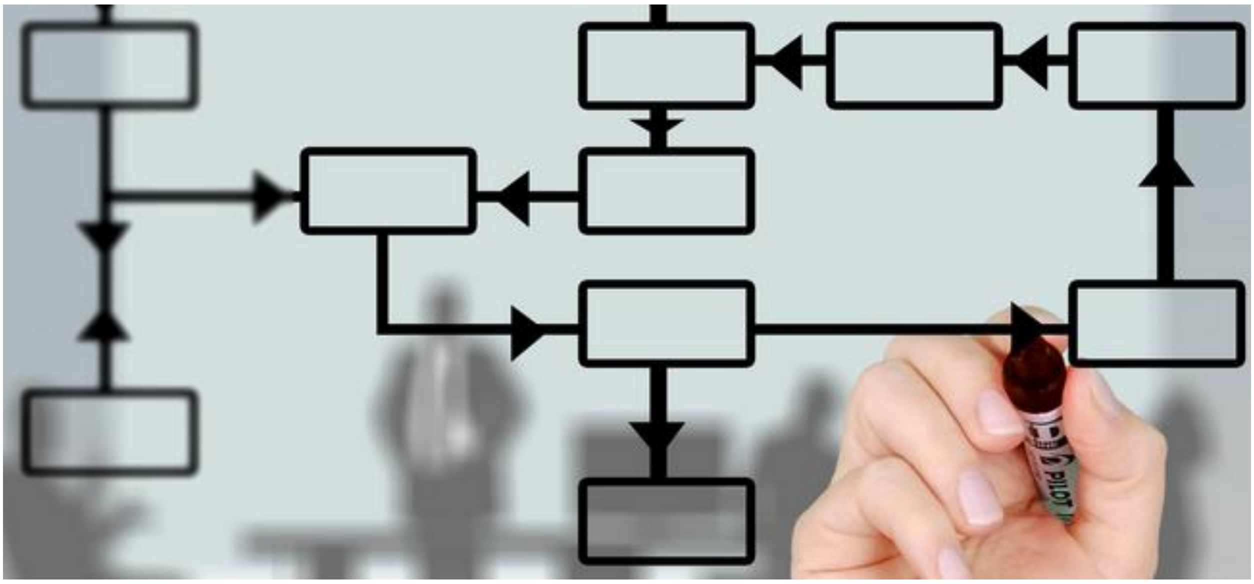
O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

QARSHI MUHANDISLIK-IQTISODIYOT INSTITUTI
Neft va gaz fakulteti

“Texnologik jarayonlarni avtomatlashtirish va boshqarish” kafedrasi

“HISOBLASH USULLARINI ALGORITMLASH” FANIDAN

O’QUV USLUBIY MAJMUA



QARSHI-2023

Tuzuvchilar:	**“Texnologik jarayonlarni avtomatlashtirish va boshqarish” kafedrasi katta o’qituvchisi F. D. Jo’rayev**
	“Texnologik jarayonlarni avtomatlashtirish va boshqarish” kafedrasi assistenti G’.Maxmatqulov
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Uslubiy qo’llanma 5311000-Texnologik jarayonlar va ishlab chiqarishni avtomatlashtirish va boshqarish (kimyo, neft-kimyo va oziq-ovqat sanoati)” ta'lim yo‘nalishi talabalari uchun “Hisoblash usullarini algoritmlash” fanidan amaliy mashg’ulotlarini bajarishda foydalanishga mo’ljallangan.

Uslubiy qo’llanma “Texnologik jarayonlarni avtomatlashtirish va boshqarish” kafedrasi yig’ilishida (№_______qaror,________), Neft va gaz fakulteti Uslubiy kengashida (№_______qaror,_______), QMII Uslubiy kengashida (№ _______qaror,______) ko’rib chiqildi va chop etishga tavsiya etildi.

© 5311000 -“TJvaIChAvaB” ta'lim yo‘nalishi talabalari uchun “Hisoblash usullarini algoritmlash” fanidan uslubiy qo’llanma. QarMII,F.D.Jo’rayev, G’.Maxmatqulov: Qarshi, 2019.-68 b.

KIRISH

Ishlab chiqarish texnika sohasida zamonaviy loyihalar, ilmiy tadqiqot ishlari jarayonida muhandislik hisoblashlari uning ajralmas muhim qismi hisoblanadi. Bunda mutaxassislar tomonidan amalgam oshiriladigan hisoblash ishlari «Hisoblash usullarini algoritmlash» fani tushunchalari bilan bevosita bog'liqdir. Shu sababli ushbu fan dasturi asosida zamonaviy o'quv adabiyotlarini yaratish dolzarb masalalardan biri sanaladi. Obektlar, hodisalar va jarayonlarni matematik modellashtirish asosida matematik modellarni kompyuterda amalga oshirish, ya'ni matematik modellar tenglamalarini sonli usullar bilan kompyuterda taqribiy yechish algoritmlarini ishlab chiqish, algoritmlarni amalga oshiruvchi dasturiy ta'minot yaratish, kompyuterda hisoblash tajribalarini o'tkazish, tajriba natijalarini tahlil qilish, xulosa chiqarish va qaror qabul qilish yotadi.

"Hisoblash usullarini algoritmlash" fani buyicha tayyorlangan mazkur qo'llanma fanning o'quv dasturi asosida tayyorlangan. Ta'lim yo'nalishi malaka talablari hisobga olingan.

Ushbu uslibiy qo'llanmada mavzularga oid amaliy masalalar yechilish uslublari bilan ularga tuzilgan algoritmlarni, hamda hisoblash natijalarini yoritadi. Shuningdek amaliy matematikada hozirgi kunda keng qo'llanilib kelinayotgan amaliy dasturlar va dasturlash texnologiyasi yushunchalari ham keltirilgan.

Uslubiy qo'llanma 5311000-"Texnologik jarayonlar va ishlab chiqarishni avtomatlashtirish va boshqarish (kimyo, neft – kimyo va oziq – ovqat sanoati)" ta'lim yo'nalishi talabalari va hisoblash matematikasi bilan qiziquvchi ommaga mo'ljallangan.

1-AMALIY ISH

Mavzu: Algebraik va transendent tenglamalarni yechishning oddiy iteratsiya, oddiy vatarlar, urinmalar (Nyuton) va kesmani teng ikkiga bo‘lish usullari va ularning algoritmi.

Ishdan maqsad: Algebraik va transendent tenglamalarni yechimini to’gri va iterasion usullar bilan olishni o’rganish.

Nazariy qism

Amaliyotda ko’pincha

$$f(x)=0 \qquad (1.1)$$

kabi tenglamalarning ildizini taqribiy hisoblab topishga to’g’ri keladi.

1.1-teorema . Aytaylik,

1) f(x) funktsiya [a,b] kesmada uzluksiz va (a,b) intervalda hosilaga ega bo‘lsin;
2) f(a)·f(b)<0, ya’ni f(x) funktsiya kesmaning chetlarida har xil ishoraga ega bo‘lsin;
3) f’(x) hosila (a,b) intervalda o‘z ishorasini saqlasin.

U holda, (1.1) tenglama [a,b] oraliqda yagona yechimga ega bo‘ladi.

f(x)=0 tenglama berilgan bo‘lsin. [*a*,*b*] kesmada u=*f*(x) funktsiya 1.1-teoremaning barcha shartlarini qanoatlantirsin.

Oraliqni teng ikkiga bo‘lish usuli. [*a*,*b*] oraliqni $x_0=(a+b)/2$ nuqta orqali ikkita teng $[a,x_0]$ va $[x_0,b]$ oraliqlarga ajratamiz. Agar $|a-x_0|\le\varepsilon$ bo‘lsa, $x=x_0$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi. Bu shart bajarilmasa, $[a,x_0]$ va $[x_0,b]$ oraliqlardan (1) tenglama ildizi joylashganini tanlab olamiz va uni $[a_1,b_1]$ deb belgilaymiz. $x_1=(a_1+b_1)/2$ nuqta yordamida $[a_1,b_1]$ oraliqni ikkita teng $[a_1,x_1]$ va $[x_1,b_1]$ oraliqlarga ajratamiz. $|a_1-x_1|\le\varepsilon$ bo‘lsa, $x=x_1$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi, aks holda $[a_1,x_1]$ va $[x_1,b_1]$ oraliqlardan (1) tenglama ildizi joylashganini tanlab olamiz va uni $[a_2,b_2]$ deb belgilaymiz. Bu oraliq uchun yuqoridagi hisoblashlar ketma-ketligini $|a_i-x_i|\le\varepsilon$ (i=2,3,4,…) shart bajarilguncha davom ettiramiz. Natijada (1) tenglamaning $x=x_i$ taqribiy yechimini hosil qilamiz.

1.1-masala. $e^x-10x-2=0$ tenglama yechimi kesmani teng ikkiga bo‘lish usulida $\varepsilon=0{,}01$ aniqlik bilan toping.

Yechish. $f(x)=e^x-10x-2$ funksiyaning 1.1-teoremaning barcha shartlarini qanoatlantiradigan aniqlanish sohasini topish lozim.Agar bu oraliq mavjud bo’lsa tenglamaga kesmani teng ikkiga bo‘lish usulini ishlatish mumkin.

Tenglama ildizini ajratish dasturini keltiramiz. Dastur natijasiga ko’ra [-0,2;-0,1] kesmani yoki [-1;0] kesmani tanlashimiz mumkin.

$e^x-10x-2=0$ tenglama ildizini ajratish dastur matni quyidagicha bo’ladi:

```
program ildizni_ajratish; uses crt;
label 1,2;
     var a,c,x,fa,fc,h:real;
             i,M,N:integer;
        function f(x:real):real;
        begin
     f:=exp(x)-10*x-2;
        end;
begin clrscr;
     2: write('a='); read(a);
      write('N='); read(N);
       write('M='); read(M);
      h:=1/N;

begin
for i:=1 to M do begin c:=a+h; fa:=f(a);fc:=f(c);
if fa*fc<0 then goto 1;
a:=c;end; end; goto 2;
 1: writeln('a=',a:5:3);
  writeln('b=', c:5:3);
    end.
```

Dastur natijasi

```
CRT - программа завершена
a=-1
N=10
M=100
a=-0.200
b=-0.100
```

1) [-1,0] oraliqni $t_0=(-1+0)/2=-0.5$ nuqta yordamida teng ikkiga bo‘lamiz. $f(t_0)=e^{-0,5}+5-2>0$, $f(-1)=8,386>0$, $f(0)=-1<0$ bo‘lganligi uchun yechim [-0.5, 0] oraliqda yotadi.

2) bu oraliqni $t_1=(-0,5+0)/2=-0,25$ nuqta yordamida teng ikkiga bo‘lamiz. $f(-1)\cdot f(-0,25)=8,386\cdot 1,279>0$ bo‘lganligi uchun yechim $[a_2; b_2]=[-0,25;0]$ oraliqda yotadi.

Aniqlik $|b_2-a_2|=0,25>2e$ etarli bo‘lmagani uchun [-0,25;0] oraliqni $t_2=\frac{0-0,25}{2}=0,125$ nuqta yordamida teng ikkiga bo‘lamiz.

3) $f(-0.125)=0.132>0$ bo‘lganligi uchun yechim $[a_3,b_3]=[-0,12; 0]$ oraliqda yotadi. Aniqlik $|a_3-b_3|=0,125>2e=0,02$ etarli bo‘lmagani uchun [-0.125,0] oraliqni $t_3=\frac{0-0,125}{2}=0,063$ nuqta yordamida teng ikkiga bo‘lamiz.

4). $f(-0,063)=-0,461$ va $f(-0,125)=0,132$ bo‘lgani uchun yechim $[a_4,b_4]$ = [-0,125; -0,063] oraliqda yotadi. $|a_4-b_4|=0,062>2e=0,02$ etarli bo‘lmaganligi uchun [-1,12; -0,063] oraliqni t_4= (- 0,125 – 0, 063)/2= - 0,094 nuqta yordamida teng ikkiga bo‘lamiz.

5). f(-0.094)=-1,841<0, f(-0,125)=0,132>0 bo‘lgani uchun yechim [-0,125; -0.094] oraliqda yotadi va t_5=(-0,125-0,094)/2=-0,1095. bu yerda $|a_5-b_5|$=0,031>2e=0,02, bo‘lgani uchun yechim [-0,125; -0,1095] oraliqda, f(-0,1095)=-0,00872<0, t_6=(-0,125-0,1095)/2=-0,11725, bundan f(-0,11725) = 0,0623, yechim [-0,1173;-0,1095] oraliqda bo‘ladi, bu yerda f(-0,11725)=0,0623, yechim [-0,1173, -0,1095] oraliqda bo‘ladi, bu yerda

$$|-0,1095 - (-0,1173)| = |0,1173 - 0,1095| = 0,008<2e=0,02$$

bo‘lgani uchun taqribiy ildiz

$$x \approx \frac{-0,1095 - 0,1173}{2} = -0,1134 \approx -0,11$$

bo‘ladi.

Quyida $e^x - 10x - 2 = 0$ tenglamani kesmani teng ikkiga bo’lish usuli bilan yechishning blok-sxemasi va ABC Pascal dasturlash tilida yozilgan dasturi keltirilgan:

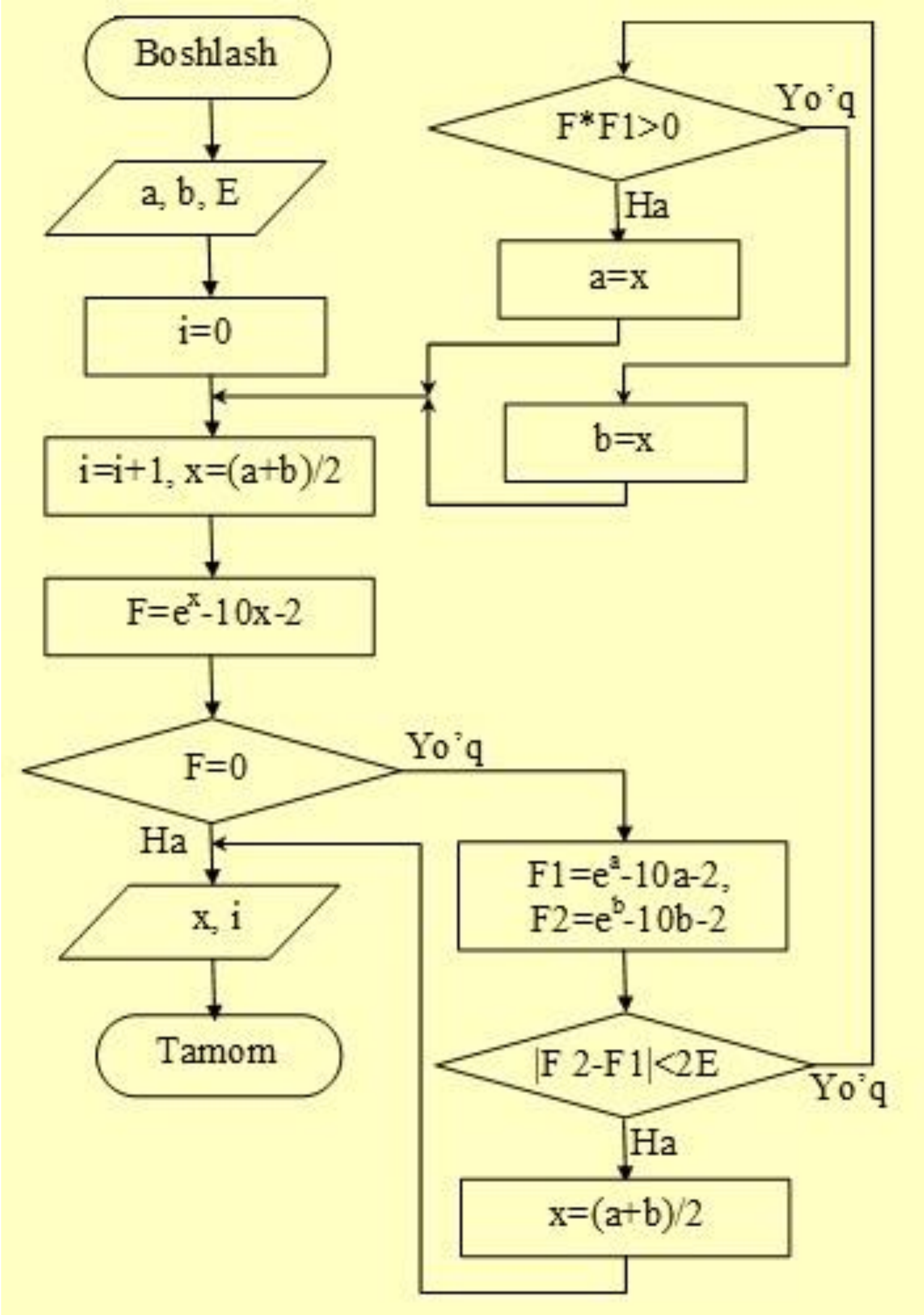
Dastur matni

```
program oraliq2; uses crt;
     var a,b,eps,x,fa,fc,c:real;
        function f(x:real):real;
        begin
     f:=exp(x)-10*x-2;
        end;
begin clrscr;
      write('a='); read(a);
      write('b='); read(b);
      write('eps='); read(eps);
      fa:=f(a);
            while abs(b-a)>eps do
            begin
            c:=(a+b)/2;
            fc:=f(c);
if fa*fc<=0 then b:=c else begin a:=c; fa:=fc end;
end;
    writeln('x=',c:10:4);
    end.
```

1.2-rasm. $e^x - 10x - 2 = 0$ tenglamasini kesmani teng ikkiga bo’lish usuli bilan yechish dasturi oynasining ko’rinishi.

Natija:

```
CRT - программа завершена
a=-1
b=0
eps=0.0001
x=    -0.1104
```



1.1-rasm. $e^x - 10x - 2 = 0$ **tenglamani kesmani teng ikkiga bo'lish usuli bilan yechishning blok-sxemasi.**

MUSTAQIL ISHLAR UCHUN TOPSHIRIQLAR

Quyidagi tenglamalar uchun:

1. Ildizlarning qisqa atrofini EHM yordamida aniqlang;
2. Aniqlangan oraliqda kesmani teng ikkiga bo‘lish usuli bilan E=0.000001 aniqlikda taqribiy hisoblang.

1). $\ln(x^2+1)-x^3-1=0$	15). $\ln(x^2+0{,}9)-2x^3-1=0$
2). $\ln(x^2+2)=x^4-3$	16). $\ln(x^2+2{,}2)=x^4-3{,}4$
3). $\ln(x^4+1)=8-x^6$	17). $\ln(x^4+1{,}5)=6{,}8-x^6$
4). $\ln(x^2+10)=10-x^4$	18). $\ln(x^2+8{,}8)=6{,}5-x^4$
5). $\ln(x^2+1)-10x^3+1=0$	19). $\ln(x^2+1{,}2)-9{,}2x^3+1{,}1=0$
6). $\ln(x^2+12)=x^4-5$	20). $\ln(1{,}2x^2+9)=1{,}3x^4-6$
7). $\ln(x^4+7)=7-x^6$	21). $\ln(x^4+4{,}7)=4{,}7-x^6$
8). $\ln(x^2+3)=4-4x+x^2$	22). $\ln(x^2+3{,}5)=1{,}3-9x+x^2$
9). $\ln(x^4+1)=2-x^2+5x$	23). $\ln(1{,}1x^4+1)=2{,}8-x^2+7x$
10). $\ln(x^2+5)=1+x^4+3x$	24). $\ln(x^2+4{,}5)=1{,}5+x^4+3{,}9x$
11). $\ln(x^2+1)-7x^3+2=0$	25). $\ln(1{,}9x^2+1)-10x^3-1=0$
12). $\ln(x^2+11)=x^4-3$	26). $\ln(1{,}3x^2+8{,}7)=2x^4-5$
13). $\ln(x^4+1)=17-x^6$	27). $\ln(x^4+6{,}1)=6{,}1-x^6$
14). $\ln(x^2+1)=4-9x+x^2$	28). $\ln(0{,}8x^2+2{,}8)=4{,}5-5x+x^2$

Chekli $[a,b]$ oraliqda aniqlangan va uzluksiz $f(x)$ funkiya berilgan bo‘lib, uning birinchi va ikkinchi tartibli hosilalari shu oraliqda mavjud bo‘lsin. Shu bilan birga $[a,b]$ da $f'(x)$ funksiya o‘z ishorasini saqlasin.

$$f(x)=0 \qquad (1)$$

tenglama $[a,b]$ oraliqda yagona yechimga ega bo‘lsin va bu yechimni berilgan $\varepsilon>0$ aniqlikda topish talab qilingan bo‘lsin. Quyida bu yechimni aniqlash uchun bir necha sonli usullar, ularning Paskal algoritmik tilida tuzilgan programmalarni keltiramiz.

Vatarlar usuli. Aniqlik uchun $f(a)>0$ ($f(a)<0$) bo‘lsin. $A=A(a;f(a))$, $B=B(b;f(b))$ nuqtalardan to‘g’ri chiziq o‘tkazamiz va bu to‘g’ri chiziqni Ox o‘qi bilan kesishish nuqtasini $x_1=a-\dfrac{b-a}{f(b)-f(a)}\cdot f(a)$ deb belgilaymiz. Agar $|a-x_1|\le\varepsilon$ bo‘lsa, $x=x_1$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi. Bu shart bajarilmasa, $b=x_1$ ($a=x_1$) deb olamiz. A, B nuqtalardan to‘g’ri chiziq o‘tkazamiz va uning Ox o‘qi bilan kesishish nuqtasini $x_2=a-\dfrac{b-a}{f(b)-f(a)}\cdot f(a)$ deb olamiz. Agar $|x_2-x_1|\le\varepsilon$ shart bajarilsa, $x=x_2$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi, aks holda $b=x_2$ ($a=x_2$) deb olib, yuqoridagi amallar ketma-ketligini $|x_i-x_{i-1}|\le\varepsilon$ (i=3,4,…) shart bajarilguncha davom ettiramiz. Natijada (1) tenglamaning $x=x_i$ taqribiy yechimini hosil qilamiz.

x_n larning ketma-ket hisoblash formulasi quyidagi ko‘rinishga ega bo‘ladi:

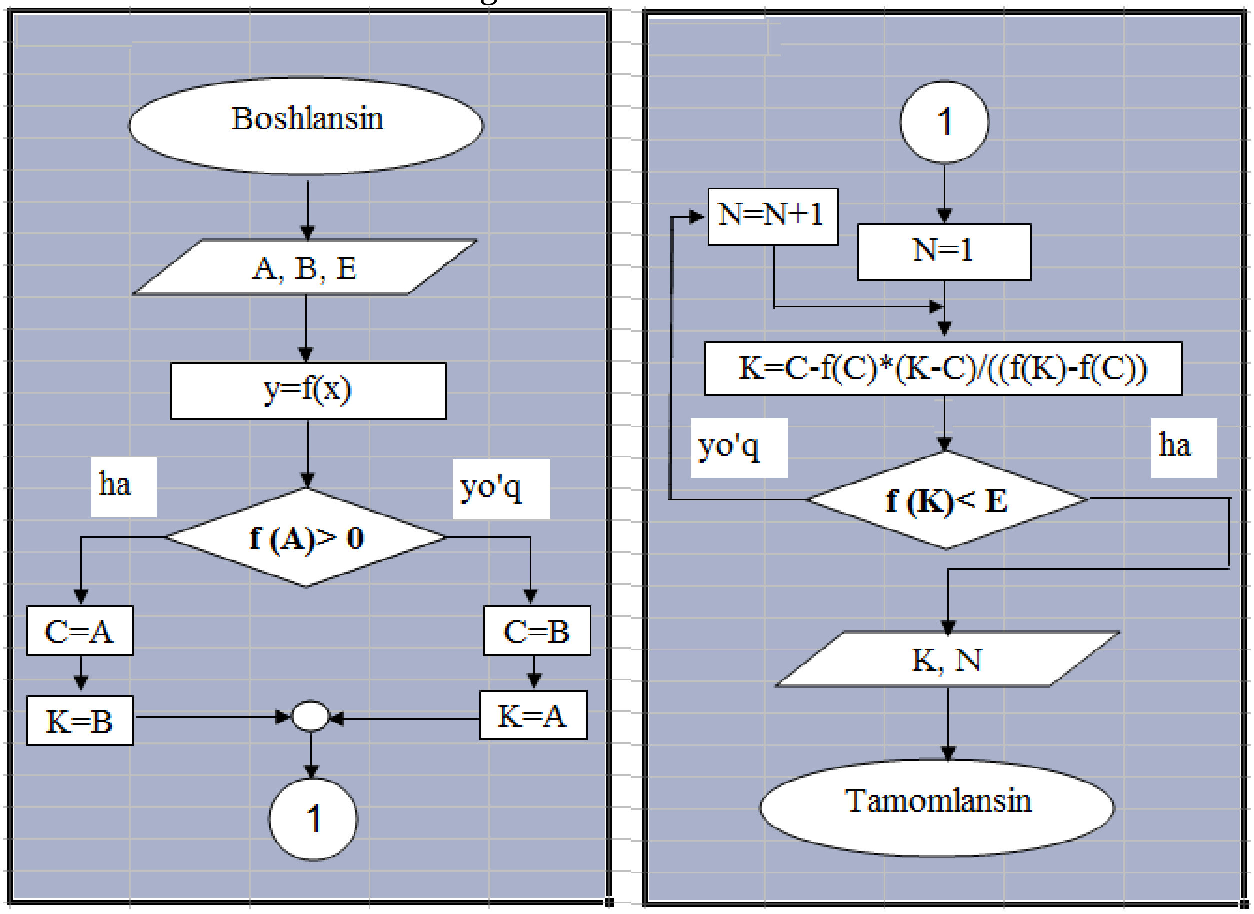
$$x_n = a - \frac{x_{n-1} - a}{f(x_{n-1}) - f(a)} \cdot f(a) \quad \left(x_n = b - \frac{x_{n-1} - b}{f(x_{n-1}) - f(b)} \cdot f(b) \right)$$

Misol. $tg(0,55x+0,1)-x^2=0$ tenglamaning [0,6;0,8] oraliqdagi ildizini ε=0,005 aniqlikda hisoblang.

Yechish. $|x_2-x_1|=0,002<\varepsilon$ bajariladi. x_2=0,7517; x_1=0,7417 bundan x=0,7517.

Berilgan tenglamaning taqribiy ildizini vatarlar usulida ε=0,0000000001 aniqlik bilan ABC Pascal dasturida hisoblaymiz.

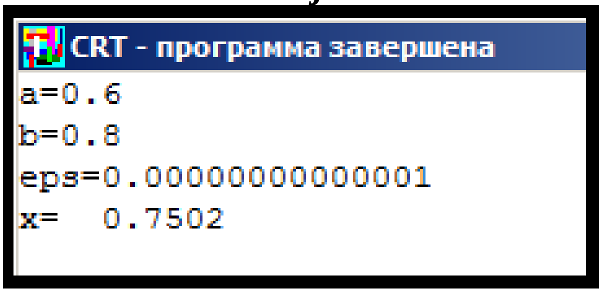
Yechish. Vatarlar usuli algoritmi blok-sxemasini keltiramiz:



Vatarlar usuliga Paskal tilida tuzilgan dasturning ko‘rinishi:

```
program vatar; uses crt;   {Vatarlar usuli}
   label 1,2;
   var a,b,eps,x:real;
    function f(x:real):real;
      begin
          f:= sin(0.55*x+0.1)/cos(0.55*x+0.1)-sqr(x);
      end;
   begin clrscr;
            write('a='); read(a);
              write('b='); read(b);
                    write('eps='); read(eps);
         2: x:=b;
             x:=b-f(b)*(b-a)/(f(b)-f(a));
             if abs(f(x))<eps then goto 1 else begin b:=x; goto 2 end;
         1: writeln('x=',x:8:4);
  end.
```

Natija:



Mustaqil yechish uchun amaliy topshiriqlar (variantlar)

Tenglamalarning vatarlar usuli yordamida 0.00000001 aniqlikda ildizini hisoblang

tenglama	kesma	tenglama	kesma
$x(5+\cos(7x-1))-4=0$	[0,8;1]	$\cos^2\left(1+\frac{1}{1+x^2}\right)-0,95x=0$	[0;0,2]
$e^{1-x}-x^5+1=0$	[1;1,2]	$x^2(3+\sin x^3)-5=0$	[1;1,2]
$x^{19}-x^2+25=0$	[-1,2;-1]	$\ln\left(1+\frac{8}{1+x^4}\right)-x^2+16=0$	[4;4,2]
$arctg(1+8x)+0,58x^5-1=0$	[0;0,2]	$x^9-x^2+16=0$	[-1,4;-1,2]
$x^{11}-\frac{1}{x^2}-1=0$	[1;1,2]	$\log_8(1+x^2)+x-1=0$	[0,6;0,8]
$\sin^2\left(1-\frac{1}{1+x^2}\right)-0,12x+0,2=0$	[7,4;7,6]	$x^7-x^2+4=0$	[-1,2;-1]
$9^{1+x}+5x=1$	[-0,6;-0,4]	$\log_5(1+x^4)+x-2=0$	[1,2;1,4]
$x^{21}-x^4+25=0$	[-1,2;-1]	$2\sin x^2+1=\frac{1}{10x^2+2}$	[1,8;2]
$2^x(5-\sin x)+x^3=0$	[-1,4;-1,2]	$x^5-x^2+9=0$	[-1,6;-1,4]
$3\cos x^2+0,5=\frac{1}{10x^2+1}$	[1,2;1,4]	$arctgx^2+0,5=\frac{1}{10x^2+1}$	[0,2;0,4]
$arctgx^2-1=\frac{1}{1+x^2}$	[1,6;1,8]	$\ln(x^2+7)-1=\frac{1}{1+x^2}$	[0,2;0,4]
$\ln(x^2+17)-7=\frac{1}{1+x^2}$	[32,8;33]	$\cos(1+x^2)-0,4=\frac{1}{x^2+5}$	[4;4,2]
$\cos(x^3-1)-0,3=\frac{1}{x^4+11}$	[-5,2;-5]	$(x^5-1)\sin(x+1)-3x=1$	[5,2;5,4]

Nyuton usuli (Urinmalar usuli). $[a,b]$ oraliqda $f'(x)$ va $f''(x)$ ning ishoralari o‘zgarmasdan qolsin. $f(x)$ funksiya grafigining $V=V(b,f(b))$ nuqtasidan urinma o‘tkazamiz. Bu urinmaning Ox o‘qi bilan kesishgan nuqtasini b_1 deb belgilaymiz. $f(x)$ funksiya grafigining $V_1=V_1(b_1,f(b_1))$ nuqtasidan yana urinma o‘tkazamiz va bu urinmaning Ox o‘qi bilan kesishgan nuqtasini b_2 deb belgilaymiz. Bu jarayonni bir necha marta takrorlab, $b_1, b_2, ..., b_n$ larni hosil qilamiz. $|b_n - b_{n-1}| < \varepsilon$ shart bajarilganda hisoblash to‘xtatiladi. $b_i = b_{i-1} - \frac{f(b_{i-1})}{f'(b_{i-1})}$

Bu usulni ikki holat uchun ko’rib chiqamiz.

1- h o l a t . Faraz qilaylik, *f(a) < 0, f(b) > 0, f'(x) > 0, f''(x) > 0* yoki *f(a)>0, f(b) < 0, f'(x) < 0, f''(x) < 0*

Urinmaning tenglamasi quyidagicha:

$$y - f(b) = f'(b)\,(x-b),$$

bu yerda $y=0$, $x=x_1$ deb , (2.1) ni x_1 nisbatan yechsak,

$$x_1 = b - \frac{f(b)}{f'(b)}$$

Shu mulohazani [a; x_1] kesma uchun takrorlab, x_2 ni topamiz:

$$x_2 = x_1 - \frac{f(x)}{f'(x)}$$

Umuman olganda $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Hisoblashni $|x_{n+1} - x_n| \leq \varepsilon$ shart bajarilganda to’xtatamiz.

2- holat. Faraz qilaylik f(a) < 0, f(b) > 0, f'(x) > 0, f''(x) < 0 yoki f(a)>0, f(b) < 0, f'(x) < 0, f''(x) > 0 . y = f(x) egri chiziqka A nuqtada urinma o’tkazamiz, uning tenglamasi: y - f(a) = f' (a) (x – a), bu yerda y=0, x=x1 decak,

$$x_1 = a - \frac{f(a)}{f'(a)}$$

[x1;b] kesmadan

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Umuman

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1-Misol. *x-sinx=0,25* tenglamaning ildizi ε=0,0001 aniqlikda urinmalar usuli bilan aniqlansin.

Y e c h i s h . Tenglamaning ildizi [0,982; 1,178] kesmada ajratilgan, bu yerda *a=0,982*; *b=1,178*;

[0,982; 1,178] kesmada *f'(x)=1-cosx x>0;* *f''(x) = sin x>0* , bo’lgani uchun 1- holat bo’yicha yechiladi. ($x_0 = b$)

[0,982; 1,178] kesmada boshlangich yaqinlashishda x_0 =1,178 olinadi. Keyingi hisoblashlarni tegishli formula vositasida bajaramiz.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{bunda } n = 0 \text{ bulsa,}$$

ya`ni: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1{,}1778 - \frac{1{,}1778 - \sin(1{,}1778) - 0{,}25}{1 - \cos(1{,}1778)} = 1{,}1715$

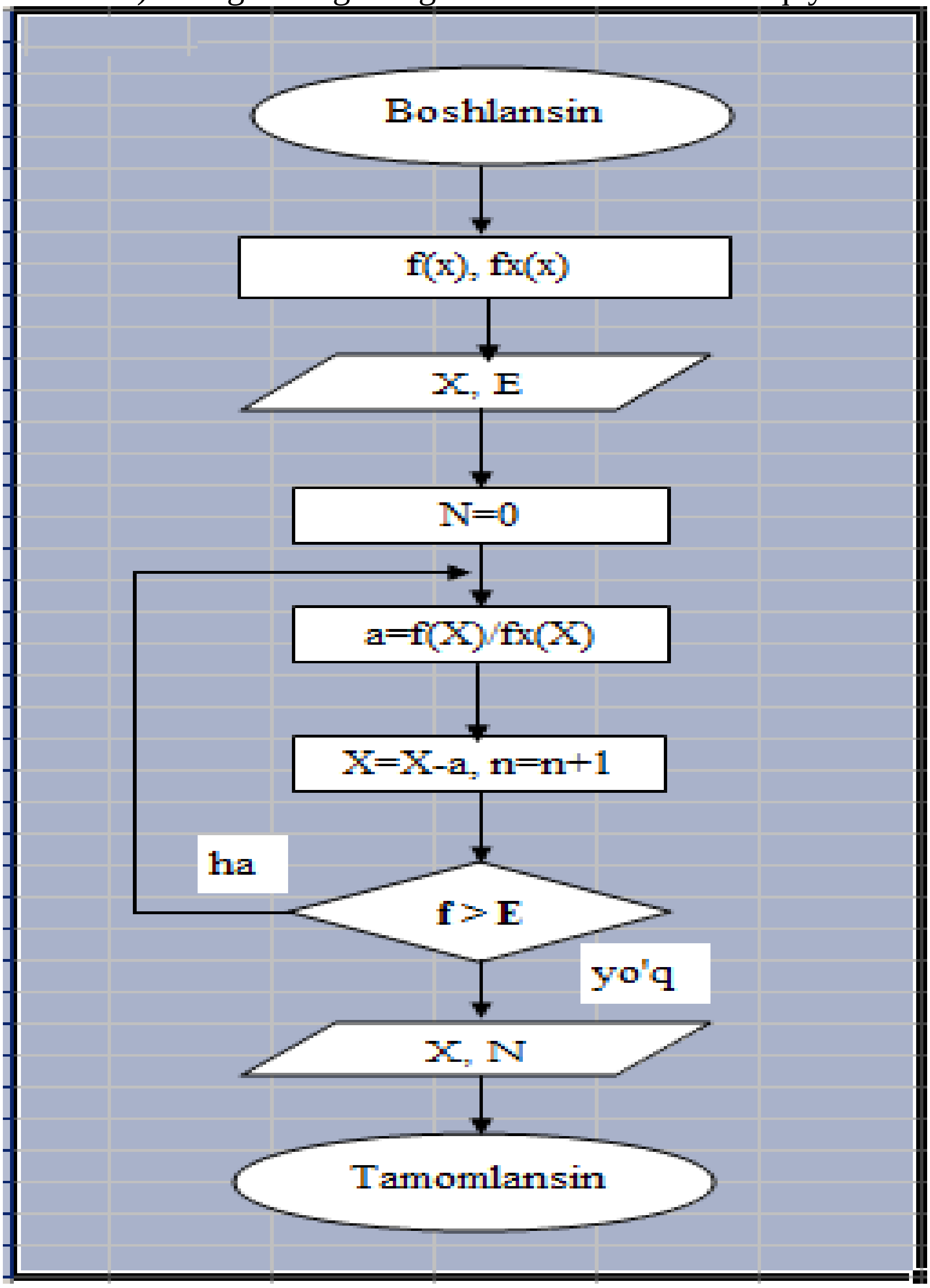
Hisoblash natijalarini quyidagi 1-jadvalda keltiramiz.

1 - jadval

n	x_n	$- \sin x_n$	$f(x_n)=x_n-\sin x_n-0{,}25$	$f'(x_n)=1-\text{sos}x_n$	$\frac{f(x_n)}{f'(x_n)}$
0	1,178	- 0,92384	0,00416	0,61723	- 0,0065
1	1,1715	- 0,92133	0,00017	0,61123	- 0,0002
2	1,1713	- 0,92127	0,00003	0,61110	- 0,0005
3	1,17125				

Jadvaldan ko'rinadiki, $x_3 - x_2 = |1{,}17125 - 1{,}1713| = 0{,}00005 < \varepsilon$. Demak yechim deb $x = 1{,}17125$ ni ($\varepsilon = 0{,}0001$ aniqlikda) olish mumkin.

Nyuton (urinmalar) usuliga tuzilgan algoritm blok – sxemasini quyida keltirilgan:



2-Misol. $tg(0,55x+0,1)-x^2=0$ tenglamaning [0,6;0,8] oraliqdagi ildizini $\varepsilon=0,005$ aniqlikda hisoblang.

Yechish. $|x_2-x_1|=0,002\le\varepsilon$; $x=x_2=0,7503$.

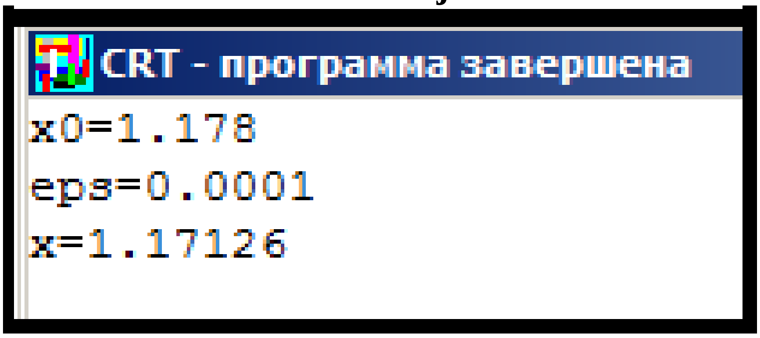
Urinmalar usuliga Paskal tilida tuzilgan dasturning ko'rinishi:

```
program urinma; uses crt;          {Urinmalar usuli}
    var x0,eps,x1,a:real; function f(x:real):real;
               begin
                    f:=      { f(x) funksiyasining ko'rinishi }
               end;
          function fx(x:real):real;
               begin
                    fx:=     { f'(x) funksiyasining ko'rinishi}
               end;
begin     clrscr;  write('x0='); read(x0);  write('eps='); read(eps);
          x1:=x0;  repeat
          a:=f(x1)/fx(x1);
          x1:=x1-a;
   until abs(f(x1))<eps;
          writeln('x=',x1:10:4);
end.
```

1-misolda berilgan tenglamaning 0,0001 aniqlikdagi ildizini ABC Pascal dasurida tuzilgan dasturda hisoblaymiz.

```
program urinma; uses crt;
        var x0,eps,x1,a:real;
function f(x:real):real;
              begin
 f:= x-sin(x)-0.25;
                end;
function fx(x:real):real;
              begin
      fx:=1-cos(x);
              end;
begin      clrscr;
                write('x0='); read(x0);
                write('eps='); read(eps);
                x1:=x0;
      repeat
                a:=f(x1)/fx(x1);
              x1:=x1-a;
      until abs(f(x1))<eps;
              writeln('x=',x1:5:5);
end.
```

Dastur natijasi:



Mustaqil bajarish uchun amaliy ish variantlari.

Berilgan tenglamalarni Nyuton usuli (Urinmalar usuli) da 0,000001 aniqlikda yeching.

Tenglama	kesma	Tenglama	kesma
$x\sin(5x+2)-x^2=-0,27$	[0,3;0,4]	$x\cos(10x-1)-x^4+1=0$	[-1;-0,9]
$e^x-e^{-x}-2=0$	[0;1]	$\frac{\ln 1,4}{3+x^2}-\sin x=0,16$	[3,2;3,3]
$3\sin\sqrt{x}+0,35x-3,8=0$	[2;3]	$x\sin(8x+1)-x^2=0,08$	[0;0,1]
$x-2+\sin\frac{1}{x}=0$	[1,2;2]	$1-x+\sin x-\ln(1+x)=0$	[0;1,5]
$x\sin(5x+8)-x^2=0,2$	[-0.5;-0.4]	$x-\frac{1}{3+\sin 3,6x}=0$	[0;0,85]
$x^2-\ln(1+x)-3=0$	[2;3]	$x\sin(11x+1)-x^4+1=0$	[0,8;0,9]
$\ln x-x+1,8=0$	[2;3]	$0,1x^2-x\ln(x+1)=0$	[1;2]
$x+\cos\left(x^{0,52}+2\right)=0$	[0,5;1]	$\sqrt{1-0,4x}-\arcsin x=0$	[0;1]
$x^2+10x-10=0$	[0;1]	$3x-4\ln x-5=0$	[2;4]
$0,4+arctg\sqrt{x}-x=0$	[1;2]	$\arccos x-\sqrt{1-0,3x^2}=0$	[0;1]
$2x-3\ln x-3=0$	[0,5;0,6]	$4+\arcsin\sqrt{x}-x=0$	[-1;5]
$0,4+tgx^2+x=0$	[-5;6]	$2-ctg\sqrt{x}-x^2=0$	[-4;5]
$0,4+\arcsin\sqrt{x}-x=0$	[-3;4]	$0,24-arcctg\sqrt{x}-x=0$	[-2;5]

Berilgan f(x)=0 tenglamani unga teng kuchli bo‘lgan x=φ(x) ko‘rinishdagi tenglamaga keltiramiz.

Teorema 2.1. Aytaylik,

1).. φ(x) funktsiya [a,b] oraliqda aniqlangan va differentsiallanuvchi bo‘lsin;

2).. φ(x) funktsiyaning hamma qiymatlari [a,b] oraliqqa tushsin;

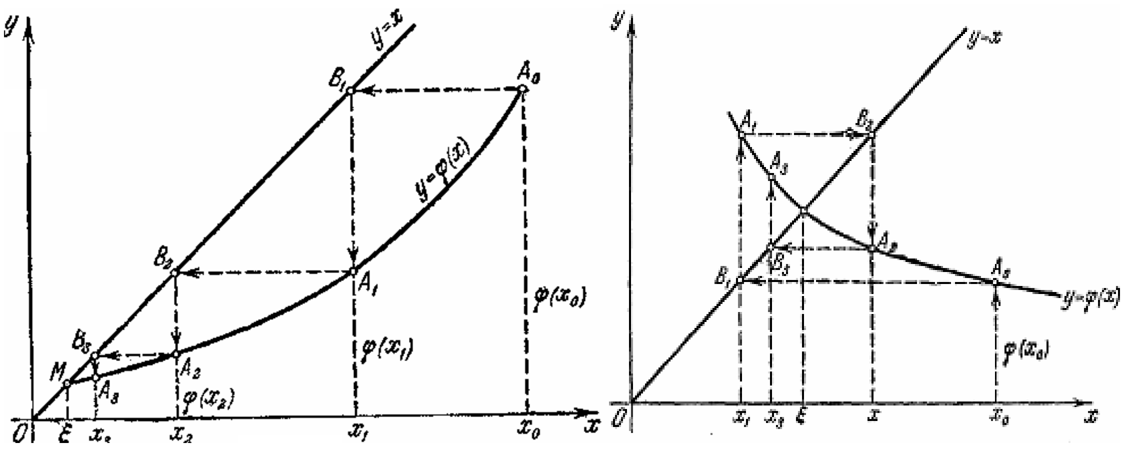
3)..[a,b] oraliqda|φ(x)|≤q<1 tengsizlik bajarilsin.

Bu holda [a,b] oraliqda x=φ(x) tenglamaning yagona x=t yechimi mavjud va bu yechim $t_0\in[a;b]$ qanday tanlanishidan qat’iy nazar

$$t_1=\varphi(t_0)\ ,\ t_2=\varphi(t_1)\ ,. \ . \ . \ ,\ t_n=\varphi(t_{n-1}),\dots$$

formulalar bilan aniqlanadigan { t_n } ketma – ketlikning limitidan iborat bo‘ladi.

Berilgan f(x)=0 tenglamani unga teng kuchli bo‘lgan x=φ(x) tenglama uchun yaqinlashish sharti bajarilganda yaqinlashish jarayonini quyidagi shakillar misolida ko‘rish mumkin.



1-Rasm.

Bu yerda, t_0 qiymat [a,b] oraliqda yotuvchi ixtiyoriy son bo‘lib, yechimning 0-yaqinlashishi, t_i – n_i yechimning i – yaqinlashishi deb yuritiladi.

Bu teorema asosida tenglama ildizini quyidagicha aniqlaymiz.

1) *f(x)*=0 tenglamaning yagona ildizi yotgan [*a,b*] kesmani biror (masalan, grafik) usul bilan aniqlaymiz.

2) [*a,b*] da *f(x)* ning uzluksizligi va $f(a)\cdot f(b)<0$ shart bajarilishini tekshiramiz.

3)Tenglamani $x=\varphi(x)$ ko‘rinishga keltirib, φ(x)∈[*a,b*] ekanligini hamda [*a;b*] da $\varphi'(x)$ mavjudligini tekshiramiz va $q=\max\limits_{x\in[a;b]}|\varphi'(x)|$ ni topamiz.

4) Agar q<1 bo‘lsa, $x_n=\varphi(x_{n-1})$ ketma-ketlikning boshlang‘ich yaqinlashishi x_0 uchun [*a;b*] ning ixtiyoriy bitta nuqtasi olamiz.

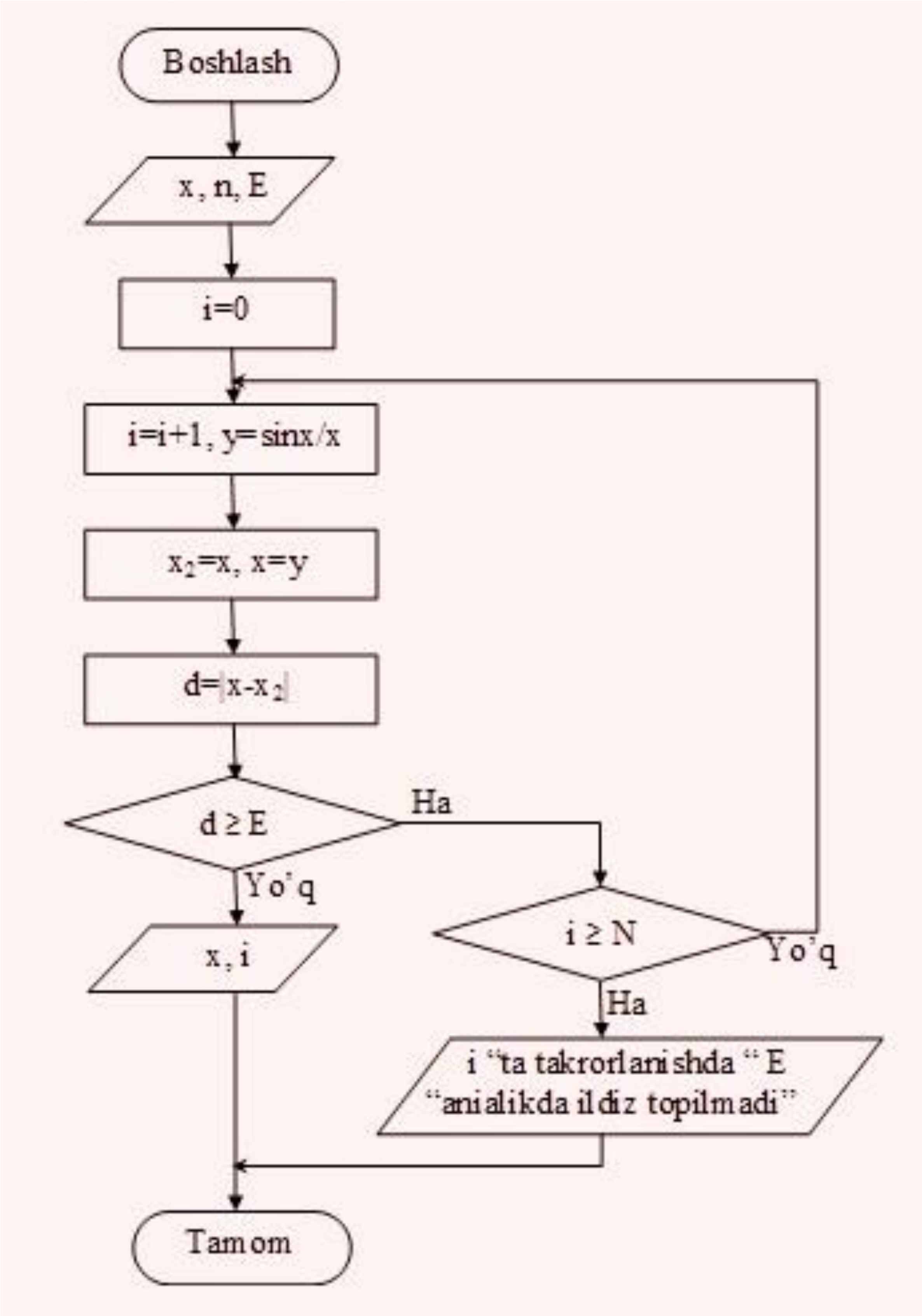
5) Ketma-ketlik hadlarini hisoblashni $|x_n - x_{n-1}| < \varepsilon\,(1-q)/q$ shart bajarilguncha davom ettiramiz.

6) Ildizning taqribiy qiymati uchun x_n ni olamiz.

Dastu matni:

```
Program   iter; uses crt;
Label  2;
Const eps = 0.00001;
VAR  x, y, del : real; n :integer;
begin
write(' Boshlangich qiymatni kiritish X0=');readln (x);
n:= 0;
2 : y:= sin(x)/x;
    del:= abs(y-x); x:=y;
    n:= n+1;
if del > eps then goto 2;
writeln('Tenglamaning taqribiy ildizi');
writeln ('x=', x);
writeln('iteratsiyalar soni n=', n);
END.
```

$\sin x - x^2 = 0$ tenglamani iteratsiya usuli bilan yechisning blok-sxemasi



Tenglama ildizini saqlovchi oraliqni topamiz. Natijada [0,8;0,9] oraliqga tegishli ihtiyoriy nuqtani boshlang'ich yechim sifatida kiritish mumkin.

```
Program   iter; uses crt;
Label  2;
Const eps = 0.00001;
VAR  x, y, del : real; n :integer;
begin
write('бошлангич кийматни киритинг X0=');readln (x);
n:= 0;
2 : y:= sin(x)/x;
     del:= abs(y-x); x:=y;
     n:= n+1;
if del > eps then goto 2;
writeln('тенгламанинг такрибий илдизи');
writeln ('x=', x);
writeln('такрорланишлар сони n=', n);
END.
```

```
CRT - программа завершена
бошлангич кийматни киритинг X0=0.8
тенгламанинг такрибий илдизи
x=0.87672408977448
такрорланишлар сони n=8
```

2-misol. Tenglamaning ildizini ajrating va uni 0,00001 aniqlikda hisoblang:

$$\frac{\ln(1+|\sin x|)}{e^x+\cos^2 x}-\frac{1}{2x^2}=arctg\frac{x}{3}$$

Yechish. Ildizni ajratamiz:

```
program ildizni_ajratish; uses crt;
label 1,2;
     var a,c,x,fa,fc,h:real;
             i,M,N:integer;
        function f(x:real):real;
        begin
     f:=ln(1+abs(sin(x)))/(exp(x)+sqr(cos(x)))-1/2*sqr(x)-arctan(x/3);
        end;
begin clrscr;
     2: write('a='); read(a);
      write('N='); read(N);
       write('M='); read(M);
      h:=1/N;

begin
for i:=1 to M do begin c:=a+h; fa:=f(a);fc:=f(c);
if fa*fc<0 then goto 1;
a:=c;end; end; goto 2;
 1: writeln('a=',a:5:3);
  writeln('b=', c:5:3);
    end.
```

```
CRT - программа завершена
a=0
N=10
M=100
a=0.100
b=0.200
```

Demak, [0,1;0,2] oraliqda tenglama ildizi mavjud va uni hisoblaymiz:

```
Program   iter; uses crt;
Label  2;
Const eps = 0.00001;
VAR  x, y, del, t : real; n :integer;
begin
write('бошлангич кийматни киритинг X0=');readln (x);
n:= 0;
2 : t:=ln(1+abs(sin(x)))/(exp(x)+sqr(cos(x)))-1/2*sqr(x);
y:= 3*sin(t)/cos(t);
     del:= abs(y-x); x:=y;
     n:= n+1;
if del > eps then goto 2;
writeln('тенгламанинг такрибий илдизи');
writeln ('x=', x);
writeln('такрорланишлар сони n=', n);
END.
```

```
CRT - программа завершена
бошлангич кийматни киритинг X0=0.1
тенгламанинг такрибий илдизи
x=0.179581940350565
такрорланишлар сони n=16
```

Mustaqil bajarish uchun topshiriqlar.

1. Tenglamalarning ildizini ajrating va uni 0,00001 aniqlikda iterasiya usulida yechish algoritm blok-sxemasini va ildizni hisoblash dasturini tuzing. Ildizni berilgan aniqlikda hisoblang.

1). $\frac{\ln\left(1+\left|0{,}2x-\cos^2 x\right|\right)}{e^x+\sin^2 x}-\frac{1}{1+2x^4}=arctg\frac{x}{K}$ *(bu yerda K=3,5,7,9,11,13,15,17)*

2). $12{,}01x^5+1{,}89x^4-0{,}98x^2+4521x=1{,}789$

3).

1.	1) $2^x+5x-3=0$ 2) $3x^4-4x^3-12x^2-5=0$ 3) $0.5^x+1=(x-2)^2$	**2.**	1) $arctgx-1/(3x^3)=0$ 2) $2x^3-9x^2-60x+1=0$ 3) $[\log_2(-x)](x+2)=-1$
3.	1) $5^x+3x=0$ 2) $x^4-x-1=0$ 3) $0.5^x+x^2=2$ 4) $(x-1)^2Ln(x+1)=1$	**4.**	1) $2e^x=2+5x$ 2) $2x^4-x^2-10=0$ 3) $xLog_3(x+1)=1$ 4) $Cos(x+0.5)=x^3$
5.	1) $3^{x-1}-2-x=0$ 2) $3x^4+8x^3+6x^2-10=0$ 3) $(x-4)^2\log_{0.5}(x-3)=-1$	**6.**	1) $arctgx-1/(2x^3)=0$ 2) $x^4-18x^2+6=0$ 3) $x^2 2^x=1$

7.	1) e^{-2x}-2x+1=0 2) $x^4+4x^3-8x^2$-17=0 3) $0.5^x-1=(x+2)^2$ 4) x^2Cos2=-1	**8.**	1) 5^x-6x-3=0 2) $x^4-x^3-2x^2$+3x-3=0 3) 0.5^x-2x^2- 3=0 4) xLog(x+1)=1
9.	1) arctg(x-1)+2x=0 2) $3x^4+4x^3-12x^2$+1=0 3) $(x-2)^2 2^x$=1 4) x^2-20Sinx=0	**10.**	1) 2arctgx-x+3=0 2) $3x^4-8x^3-18x^2$+3=0 3) 2Sin(x+1.1)=$0.5x^2$-1 4) 2Logx-x/2+1=0
11	1) 3^x+2x-2=0 2) $2x^4-8x^3+8x^2$-1=0 3) $[(x-2)^2-1]2^x$ =1 4) (x-2)Cosx=1	**12.**	1) 2arctgx-3x+2=0 2) $2x^4+8x^3+8x^2$ -1=0 3) Sin(x-0.5)-x+0.8=0 4) (x-1)Log_2(x+2)=1
13.	1) 3^x+2x-5=0 2) $x^4-4x^3-8x^2$+1=0 3) 0.5^x+x^2- 3=0 4) $(x-2)^2$Lg(x+1)=1	**14.**	1) $2e^x$+3x+3x+1=0 2) $3x^4+4x^3-12x^2$-5=0 3) Cos (x+0.3)=x^2 4) xLog_3(x+1)=2
15.	1) 3^{x-1}-4-x=0 2) $2x^3-9x^2$-60x+1=0 3) $(x-3)^2 Log_{0.5}$(x-2)=-1 4) Sinx=x-1	**16.**	1) arctgx-1/($3x^3$)=0 2) x^4-x–1=0 3) $(x-1)^2 2^x$=1 4) tg^3x=x-1
17.	1) e^x+x+1=0 2) $2x^4-x^2$-1=0 3) $0.5^x–3=(x+2)^2$ 4) $(x-2)^2 2^x$=1	**18.**	1) 3^x-2x+5=0 2) $3x^4+8x^3+6x^2$-10=0 3) $2x^2-0.5^x$=0 4 xLg(x+1)=1
19.	1) arctg(x-1)+3x-2=0 2) x^4-18x^2+6=0 3) x^2-20Sinx=0	**20.**	1) 2arctgx-x+3=0 2) $x^4+4x^3-8x^2$ -17=0 4) 2Lgx-x/2+1=0
21	1) 2^x-3x-2=0. 2)$x^4-x^3-2x^2$ +3x- 3=0; 3)$(0.5)^x$+ 1=$(x-2)^2$ 4)(x-3)cosx = -1, -2’<x<2’.	**22**	1) arctgx+2x-1=0 2) $3x^4+4x^3-12x^2$ +1=0 3)(x+2)Log_2(x)=1 4) Sin(x+1)=0.5x
23	1) 3^x+2x-3=0. 2) $3x^4-8x^3-18x^2$ +2=0; 3) $(0.5)^x=4-x^2$ 4) $(x+2)^2$Lg(x+11)=1	**24**	1) $2e^x$-2x-3=0. 2) $3x^4+4x^3-12x^2$ -5=0; 3) xLog_2(x+1)=1 4) cos(x+0.5)= x^3
25	1) 3^x+2+x=0. 2) $2x^3-9x^2$ -60x+1=0; 3) $(x-4)^2 Log_{0.5}$(x-3)=-1 4) 5Sinx=x-0.5	**26**	1) arctg(x-1)+2x-3=0 2) x^4x -1=0; 3) $(x-1)^2 2^x$=1 4) x^2-10sinx = 0
27	1) $2e^x$-2x-3=0. 2) $2x^4-x^2$ -10=0; 3) $(0.5)^x-3=-(x+1)^2$ 4) x^2cos2x = 1	**28**	1) 3^x-2x-5=0. 2) $3x^4+8x^3+6x^2$ -10=0; 3) $2x^2-0.5^x$ - 3=0 4) xLg(x+1)=1

O‘z-o‘zini tekshirish uchun savollar

1. Yechim yotgan Kesmani aniqlash.
2. Boshlang‘ich shartni tanlash usulini tushuntiring.
3. Iteratsiya usulining yaqinlashish shartini ayting.
4. Iteratsiya usulida boshlang‘ich shartni tanlash usulini tushuntiring.
5. Kesmani ikkiga bo‘lish usuli va uning yaqinlashish shartini ayting.
6. Tenglamalarni taqribiy hisoblashda ketma-ket yaqinlashish (iteratsiya) shartlari.
7. Iteratsiya usulini qo‘llashda x=j(x) tenglamadagi j(x) uchun qo‘yilgan shartlar.
8. Iteratsiya usulini qo‘llashda x=j(x) tenglamadagi j‘(x) uchun qo‘yilgan shartlar.
9. Ketma-ket yaqinlashish (iteratsiya) usulida boshlang‘ich yaqinlashish qiymatini tanlash qoidasi.
10. Kesmani ikkiga bo‘lish usuli va uni qo‘llash haqidagi shartlar.
11. Iteratsiya usulining mohiyatini aytib bering.
12. Tenglama iteratsiya usulini qo’llash uchun qanday ko’rinishga olib keltiriladi.
13. Iteratsiya usulining yaqinlashish shartining ma’nosini aytib bering.
14. Iteratsiya usulining nazariy va amaliy xatoliklarining ma’nosini aytib bering.
15. Vatarlar usulida yaqinlashish shartini tushuntiring?
16. Vatarlar usulining yaqinlashish formulasini keltirib chiqaring?
17. Algoritmni blok-sxemada tasvirlash afzalligi va kamchiligi nimada?

2-AMALIY ISH

Mavzu: Chiziqli algebraik tenglamalar sistemasini yechishning Gauss, oddiy iteratsiya, Zeydel usullari va ularning algoritmi

Ishning maqsadi : Talabalarga chiziqli algebraik va transendent tenglamalar sistemasini Gauss, usulida yechish algoritmlarini berish hamda bu usullarga Paskal tilida tuzilgan dasturda ishlashga o‘rgatish.

Nazariy qism

Bizga n ta noma’lumli n ta chiziqli algebraik tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n} = b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n} = b_2 \\ .... \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n \end{cases} \tag{1}$$

berilgan bo‘lsin. Bu yerda a_{ij}, b_i lar berilgan sonlar, x_i lar noma’lumlar (i,j=1,2,...,n). Agar (1) sistemaga mos keluvchi asosiy determenant 0 dan farqli, ya’ni

$$\begin{vmatrix} a_{11}\ a_{12}\ ...\ a_{1n} \\ a_{21}\ a_{22}\ ...\ a_{2n} \\ .... \\ a_{n1}\ a_{n2}\ ...\ a_{nn} \end{vmatrix} \neq 0$$

bo‘lsa u yagona yechimga ega bo‘ladi.

Chiziqli algebraik tenglamalar sistemasini yechishning bir necha usullari mavjud bo‘lib, ulardan asosiylari Kramer, Gauss, teskari matritsa, iteratsiya usullaridir. Bu usullardan Gauss usuli bilan yechish algoritmini (1) sistema uchun ko‘rib chiqaylik.

Gauss usuli. Gauss usuli yoki no’malumlarni ketma-ket yo‘qotish usuli chiziqli algebraik tenglamalar sistemasini aniq yechish usuli hisoblanadi. Bu usulining algoritmi quyidagi hisoblashlar ketma-ketligidan iborat.

$a_{11} \neq 0$ bo‘lsin (agar $a_{11} = 0$ bo‘lsa, sistemadagi tenglamalarning o‘rnini almashtirib $a_{11} \neq 0$ ga ega bo‘lish mumkin). (1) sistemadagi birinchi tenglamaning barcha hadlarini a_{11} ga bo‘lib

$$x_1 + a_{12}^{(1)}x_2 + ... + a_{1n}^{(1)}x_n = b_1^{(1)}$$

ni hosil qilamiz. Bu tenglamani ketma-ket $a_{21}, a_{31}, ..., a_{n1}$ larga ko‘paytirib, undan sistemaning keyingi tenglamalarini ayiramiz va

$$\begin{cases} x_1 + a_{12}^{(1)} x_2 + ... + a_{1n}^{(1)} x_n = b_1^{(1)} \\ a_{22}^{(1)} x_2 + ... + a_{2n}^{(1)} = b_2^{(1)} \\ .... \\ a_{n2}^{(1)} x_2 + ... + a_{nn}^{(1)} x_n = b_n^{(1)} \end{cases} \qquad (2)$$

sistemaga ega bo‘lamiz. Bu yerda $a_{ij}^{(1)} = a_{ij} - \frac{a_{i1} a_{1j}^{(1)}}{a_{11}}$, $b_j^{(1)} = b_j - \frac{b_1 a_{j1}}{a_{11}}$ i=2,…,n; j=2,3,…,n.

(2) sistema uchun yuqoridagi hisoblashlar (noma’lumlarni ketma-ket yuqotish) ni bir necha bor takrorlab, quyidagi

$$\begin{cases} x_1 + a_{12}^{(1)} x_2 + a_{13}^{(1)} x_3 + ... + a_{1n}^{(1)} x_n = b_1^{(1)} \\ x_2 + a_{23}^{(2)} x_3 + ... + a_{2n}^{(2)} x_n = b_2^{(2)} \\ .... \\ x_n = b_n^{(n)} \end{cases} \qquad (3)$$

sistemani hosil qilamiz va x_i larni topish uchun

$$x_k = a_{k,n-1}^{(k-1)} - \sum_{j:=k+1}^{n} a_{kj}^{(k-1)} x_j ,k = n, n-1, n-2,...,1$$

formulaga ega bo‘lamiz.

Chiziqli algebraik tenglamalar sistemasini Gauss usulida yechish uchun Paskal algoritmik tilida tuzilgan dastur matni.

```
Program gauss; uses crt;
const n=4;      {tenglamalar soni}
type
stroka=array[1..n] of real;
matrisa=array[1..n, 1..n+1] of real;
vektor=array[1..n] of real;
var
a:matrisa; x:vektor; max,c:real;
i,j,k,m:integer;
procedure gauss_1(b:matrisa; var y:vektor);
begin
             for i:=1 to n do
               begin          max:=abs(b[i,i]); j:=i;
      for k:=i+1 to n do if abs(b[k,i])>max then
               begin max:=abs(b[k,i]);
                 j:=k; end;
                 if j<>i then for k:=i to n+1 do
      begin c:=b[i,k]; b[i,k]:=b[j,k];
                 b[j,k]:=c; end;
                   c:=b[i,i];        for k:=i to n+1 do b[i,k]:=b[i,k]/c;
```

for m:=i+1 to n do
begin c:=b[m,i];
*for k:=i+1 to n+1 do b[m,k]:=b[m,k]-b[i,k]*c;*
end; end;
y[n]:=b[n,n+1];
for i:=n-1 downto 1 do
begin
y[i]:=b[i,n+1];
*for k:=i+1 to n do y[i]:=y[i]-b[i,k]*y[k]*
end; end;
begin clrscr;
for i:=1 to n do
for j:=1 to n+1 do
begin
write('a[',i:1,',',j:1,']=');
read(a[i,j]); end;
gauss_1(a,x);
writeln('Sistemaning yechimi');
for i:=1 to n do writeln('x[',i:1,']=',x[i]:10:4); end.

Misol. Berilgan chiziqli algebraik tenglamalar sistemalarini Gauss usuli yordamida yeching.

$$\begin{cases} 5x_1 - x_2 + x_3 - 7x_4 = 10 \\ 2x_1 - 8x_2 - 31x_4 = 22 \\ 3x_1 - 11x_3 + x_4 = -33 \\ 2x_1 + 2x_2 - 2x_3 - 50x_4 = -60 \end{cases}$$

Yechish. Berilgan dastur matnidan foydalanib hisoblaymiz :

```
program gauss; uses crt;
const n=4;      {tenglamalar soni}
type   stroka=array[1..n] of real;
matrisa=array[1..n, 1..n+1] of real; vektor=array[1..n] of real;
var a:matrisa; x:vektor; max,c:real; i,j,k,m:integer;
procedure gauss_1(b:matrisa; var y:vektor);
begin for i:=1 to n do begin max:=abs(b[i,i]); j:=i;
for k:=i+1 to n do if abs(b[k,i])>max then  begin max:=abs(b[k,i]);
j:=k; end; if j<>i then for k:=i to n+1 do begin c:=b[i,k]; b[i,k]:=b[j,k];
b[j,k]:=c; end; c:=b[i,i];for k:=i to n+1 do b[i,k]:=b[i,k]/c;
for m:=i+1 to n do begin c:=b[m,i];   for k:=i+1 to n+1 do
b[m,k]:=b[m,k]-b[i,k]*c;  end;   end;  y[n]:=b[n,n+1];  for i:=n-1 downto 1 do
 begin  y[i]:=b[i,n+1]; for k:=i+1 to n do y[i]:=y[i]-b[i,k]*y[k]
end; end;  begin  clrscr; for i:=1 to n do for j:=1 to n+1 do begin
 write('a[',i:1,',',j:1,']=');  read(a[i,j]); end;  gauss_1(a,x);
writeln( 'Sistemaning yechimi' );  for i:=1 to n do
writeln('x[',i:1,']=',x[i]:10:4); end.
```

Dastur natijasi

```
CRT - программа завершена
a[1,2]=-1
a[1,3]=1
a[1,4]=-7
a[1,5]=10
a[2,1]=0
a[2,2]=2
a[2,3]=-8
a[2,4]=-31
a[2,5]=22
a[3,1]=3
a[3,2]=0
a[3,3]=-11
a[3,4]=1
a[3,5]=-33
a[4,1]=2
a[4,2]=2
a[4,3]=-2
a[4,4]=-50
a[4,5]=-60
Sistemaning yechimi
x[1]= -502.2581
x[2]=-2008.9355
x[3]= -142.4839
x[4]=  -93.5484
_
```

Topshiriq

1-masala. Berilgan chiziqli algebraik tenglamalar sistemalarini Gauss usuli yordamida yeching.

1. $\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 1 \\ 2x_1 - x_2 - 3x_4 = 2 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 - 5x_4 = -6 \end{cases}$

2. $\begin{cases} x_1 + 5x_2 = 2 \\ 2x_1 - x_2 + 3x_3 + 2x_4 = 4 \\ 3x_1 - x_2 - x_3 + 2x_4 = 6 \\ 3x_1 - x_2 + 3x_3 - x_4 = 6 \end{cases}$

3. $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$

4. $\begin{cases} x_1 - 4x_2 = 2 \\ x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 2x_1 + 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$

5. $\begin{cases} 5x_1 + x_2 - x_4 = 9 \\ 3x_1 - 3x_2 - x_3 + 4x_4 = -1 \\ 3x_1 - 2x_3 + x_4 = -16 \\ x_1 - 4x_2 + x_4 = 0 \end{cases}$

6. $\begin{cases} 5x_1 - x_2 + x_3 + 3x_4 = -4 \\ x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ 2x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + x_2 + x_3 + 2x_4 = 4 \end{cases}$

7. $\begin{cases} 2x_1 + x_3 + 4x_4 = 9 \\ x_1 + 2x_2 - x_3 + x_4 = 8 \\ 2x_1 + x_2 + x_3 + x_4 = 5 \\ x_1 - x_2 + 2x_3 + x_4 = -1 \end{cases}$

8. $\begin{cases} 4x_1 - 2x_2 + x_3 - 4x_4 = 3 \\ 2x_1 - x_2 + x_3 - x_4 = 1 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6 \end{cases}$

9. $\begin{cases} 2x_1 - 6x_2 + 2x_3 + 2x_4 = 12 \\ x_1 + 3x_2 + 5x_3 + 7x_4 = 12 \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0 \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4 \end{cases}$

10. $\begin{cases} 2x_1 - x_3 - 2x_4 = -1 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = 1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$

11. $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 - x_2 - x_3 - 2x_4 = -4 \\ 2x_1 + 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$

12. $\begin{cases} x_1 + 5x_2 + 3x_3 - 4x_4 = 20 \\ 3x_1 + x_2 - 2x_3 = 9 \\ 5x_1 + 7x_2 + 10x_4 = -9 \\ 3x_2 - 5x_3 = 1 \end{cases}$

13. $\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4 \\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8 \end{cases}$

14. $\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8 \\ x_1 - 3x_2 - 6x_4 = 9 \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0 \end{cases}$

15. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$

16. $\begin{cases} 3x_1 + 2x_2 - 2x_3 + 4x_4 = 7 \\ 2x_1 - x_2 + 2x_3 - 3x_4 = -5 \\ -x_1 + 2x_2 - x_3 + 12x_4 = 1 \\ -7x_1 + x_2 + 2x_3 + x_4 = 3 \end{cases}$

Noma'lumlar soni ko'p bo'lganda chiziqli tenglamalar sistemasini yechishning Kramer, Gauss, teskari matrisa usullari bilan olinishi ancha murakkab bo'lib qoladi. Bunday hollarda taqribiy sonli usullardan foydalanish ancha samarali hisoblanadi. Shunday usullardan biri oddiy iterasiya usulidir.

Quyidagi tenglamalar sistemasi berilgan bo'lsin.

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \; i = 1,2,...,n \qquad (6.1)$$

Bu sistema matrisa ko'rinishida quyidagicha yoziladi:

$$Ax = b,$$

Bu yerda

$$A = \begin{pmatrix} a_{11} & a_{12} & .. & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}, \; B = \begin{pmatrix} b_1 \\ b_2 \\ ... \\ b_n \end{pmatrix}, \;\; x = \begin{pmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{pmatrix}.$$

(6.1) da $a_{ii} \neq 0$ $(i=1,n)$ deb faraz qilamiz.

Tenglamalar sistemasida 1-tenglamani x_1 ga nisbatan, 2- tenglamani x_2 ga nisbatan, va ohirgisini x_n ga nisbatan yechamiz:

$$\begin{cases} x_1 = \beta_1 + 0 + \alpha_{12}x_2 + \alpha_{13}x_3 + ... + \alpha_{1n}x_n \\ x_2 = \beta_2 + \alpha_{21}x_2 + 0 + \alpha_{23}x_3 + ... + \alpha_{2n}x_n \\ ...... \\ x_n = \beta_n + \alpha_{n1}x_2 + \alpha_{n2}x_2 + \alpha_{n3}x_3 + ... + \alpha_{nn-1}x_{n-1} + 0 \end{cases} \qquad (6.2)$$

Ushbu

$$\alpha = \begin{pmatrix} 0 & \alpha_{12} & ... & \alpha_{in} \\ \alpha_{21} & 0 & ... & \alpha_{2n} \\ ... & ... & ... & ... \\ \alpha_{n1} & a_{n2} & ... & 0 \end{pmatrix} \quad \text{va} \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ ... \\ \beta_n \end{pmatrix}$$

Matrisalar yordamida (6.2) ni quyidagicha yozish mumkin: $x = \beta + \alpha x$ (6.3)

(6.3) sistemani ketma-ket yaqinlashish usuli bilan yechamiz.

$$x^{(0)}=\beta, \quad x^{(1)} = \beta + \alpha x^{(0)}, \ x^{(2)} = \beta + \alpha x^{(1)}, ...$$

Bu jarayonni quyidagicha ifodalaymiz:

$$x^{(k)} = \beta + \alpha x^{(k-1)}, \ x^{(0)}=\beta \qquad (6.4)$$

Bu ketma-ketlikning limiti, agar u mavjud bo'lsa (6.1) sistemaning yechimi bo'ladi.

Biz

$$x^{(k)} = \begin{pmatrix} x_1^{(k)} \\ x_2^{(k)} \\ ... \\ x_n^{(k)} \end{pmatrix}$$

belgilashni kiritamiz. Agar ihtiyoriy $\varepsilon>0$ uchun $\left|x_i^{(k+1)} - x_i^{(k)}\right| < \varepsilon$ tengsizlik barcha i =1,2,...n uchun bajarilsa $x^{(k+1)} = \left(x_1^{(k+1)}, x_2^{(k+1)}, ..., x_n^{(k+1)}\right)$ мулещк (6,1) sistemaning ε aniqlikdagi yechimi deb yuritiladi.

Teorema. Agar keltirilgan (6.2) system uchun $\sum_{j=1}^{n}\left|\alpha_{ij}\right| < 1$ yoki $\sum_{i=1}^{n}\left|\alpha_{ij}\right| < 1$ shartlardan birortasi bajarilsa, uholda (6.4) iterasiyon jarayon boshlang'ich yaqinlashishni tanlashga bog'liq bo'lmagan holda yagona yechimga yaqinlashadi.

Natija (6.4) tenglamalar sistemasi uchun

$$\sum_{\substack{j \neq 1 \\ j=1}}^{n} |a_{ij}| < |a_{11}|, \ \sum_{\substack{j \neq 2 \\ j=1}}^{n} |a_{2j}| < |a_{22}|, \ ..., \ \sum_{\substack{j \neq n \\ j=1}}^{n} |a_{nj}| < |a_{nn}|$$

tengsizliklar bajarilsa (6.4) iterasiya yaqinlashuvchi bo'ladi.

Misol. Tenglamalar sistemasini ε=0,001 aniqlikda oddiy iterasiya usuli bilan yeching:

$$\begin{cases} 4x_1 + 0{,}24x_2 - 0{,}08x_3 = 8 \\ 0{,}09x_1 + 3x_2 - 0{,}15x_3 = 9 \\ 0{,}04x_1 - 0{,}08x_2 + 4x_3 = 20 \end{cases}$$

Yechish:

$$\left.\begin{array}{l} 0{,}24 + |-0{,}08| = 0{,}32 < |a_{11}| = 4 \\ 0{,}09 + |-0{,}15| = 0{,}24 < |a_{22}| = 3 \\ 0{,}04 + |0{,}08| = 0{,}12 < |a_{33}| = 4 \end{array}\right\}$$

Demak, iterasiya yaqinlashuvchi.

$$\begin{cases} x_1 = 2 - 0{,}06x_2 + 0{,}02x_3 \\ x_2 = 3 - 0{,}03x_1 + 0{,}05x_3 \\ x_3 = 5 - 0{,}01x_1 + 0{,}02x_2 \end{cases}.$$

Nolinchi yaqinlashish: $x^{(0)} = \beta = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, $x_1^{(0)} = 2,\ x_2^{(0)} = 3,\ x_3^{(0)} = 5$.

$$\alpha = \begin{pmatrix} 0 & -0{,}06 & 0{,}02 \\ -0{,}03 & 0 & 0{,}05 \\ -0{,}01 & 0{,}02 & 0 \end{pmatrix}$$

(6.4) formula yordamida hisoblashlarni bajaramiz.

$$x^{(1)} = \beta + \alpha x^{(0)} = \begin{pmatrix} 1{,}92 \\ 3{,}19 \\ 5{,}04 \end{pmatrix};\ \ x_1^{(1)} = 1{,}92;\ x_2^{(1)} = 3{,}19;\ x_3^{(0)} = 5{,}04.$$

$$x^{(2)} = \beta + \alpha x^{(1)} = \begin{pmatrix} 1{,}9094 \\ 3{,}1944 \\ 5{,}0446 \end{pmatrix};\quad x^{(3)} = \beta + \alpha x^{(2)} = \begin{pmatrix} 1{,}90923 \\ 3{,}19495 \\ 5{,}04485 \end{pmatrix};$$

$$x_1^{(2)} = 1{,}9094;\ x_2^{(2)} = 3{,}1944;\ x_3^{(2)} = 5{,}0446.$$

Ushbu jadval hosil bo'ladi.

Yaqinlashishlar (k)	x_1	x_2	x_3	$x_1^{(k)} - x_1^{(k-1)}$	$x_2^{(k)} - x_2^{(k-1)}$	$x_3^{(k)} - x_3^{(k-1)}$
0	2	3	5	-	-	-
1	1,92	3,19	5,04	0,08	0,19	0,04
2	1,9094	3,1944	5,0446	0,0106	0,0044	0,0046
3	1,90923	3,19495	5,04485	0,00017	0,00055	0,00025

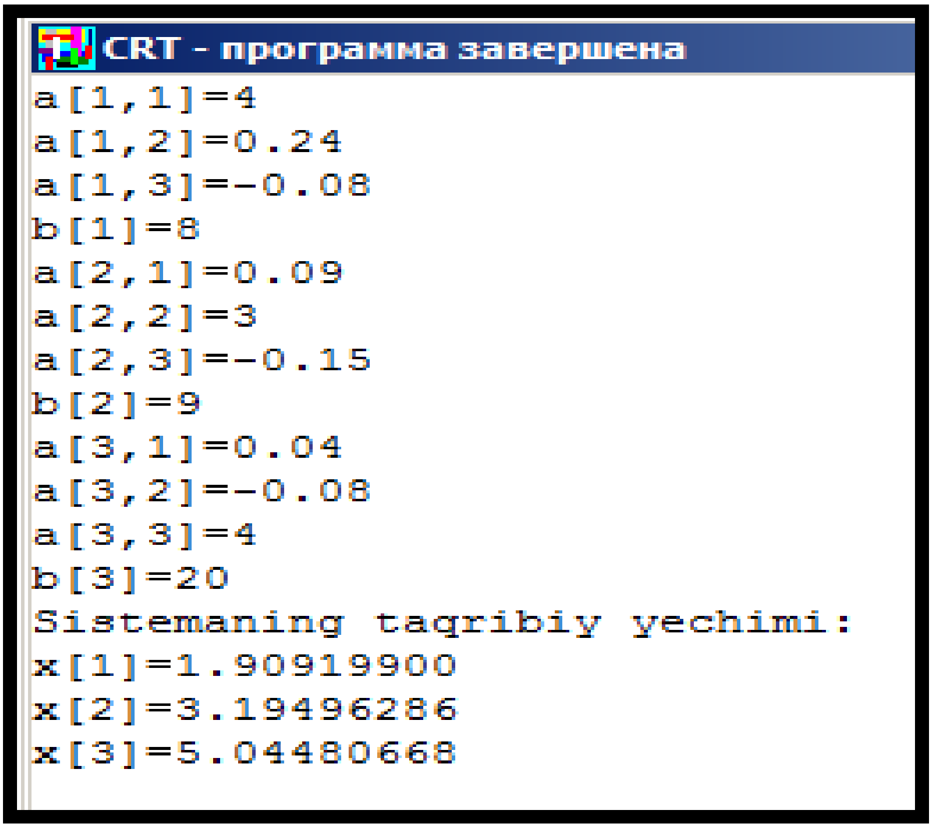
Bunda $\left|x_1^{(3)} - x_1^{(2)}\right| = 0{,}00017 < \varepsilon$, $\left|x_2^{(3)} - x_2^{(2)}\right| = 0{,}00055 < \varepsilon$, $\left|x_3^{(3)} - x_3^{(2)}\right| = 0{,}00025 < \varepsilon$ bajariladi. $x = x^{(3)}$ ChTS ning taqribiy ildizi.

Tenglamalar sistemasini oddiy iterasiya usulida yechish uchun ABC Pascal algortmik tilida tuzilgan dastur matni.

```
program iter_sis; uses crt;
 label 1,2;   const n=3; {tenglamalar coni}
      type           matrisa=array[1..n,1..n] of real;
         vektor=array[1..n] of real;
      var    a,a1:matrisa; x,x0,b,b1:vektor; eps,s:real; i,j,k:integer;
 begin      clrscr;
     for i:=1 to n do begin
     for j:=1 to n do begin     write('a[',i:1,',',j:1,']='); read(a[i,j]) end;
                          write('b[',i:1,']='); read(b[i]);   end;
     eps:=0.0001;      for i:=1 to n do begin
                          b1[i]:=b[i]/a[i,i];
                          for j:=1 to n do a1[i,j]:=-a[i,j]/a[i,i] end;
                          for i:=1 to n do begin
                                     x0[i]:=b1[i];    a1[i,i]:=0;      end;
         2: for i:=1 to n do   Begin      s:=0.0;
                              for j:=1 to n do s:=s+a1[i,j]*x0[j];
                              x[i]:=b1[i]+s;   end;          k:=0;
          for i:=1 to n do if abs(x[i]-x0[i])<eps
              then begin k:=k+1; if k=n then goto 1 end
              else begin for j:=1 to n do x0[j]:=x[j]; goto 2 end;
    1:  writeln('Sistemaning taqribiy yechimi:');
     for i:=1 to n do writeln('x[',i:1,']=',x[i]:10:8);
end.
```

```
program iter_sis; uses crt;
  label 1,2;
  const n=3;    type
            matrisa=array[1..n,1..n] of real;
            vektor=array[1..n] of real;
  var  a,a1:matrisa; x,x0,b,b1:vektor; eps,s:real; i,j,k:integer;
  begin    clrscr;
       for i:=1 to n do begin
       for j:=1 to n do begin
  write('a[',i:1,',',j:1,']='); read(a[i,j]) end;
  write('b[',i:1,']='); read(b[i]);  end;   eps:=0.0001;
for i:=1 to n do begin      b1[i]:=b[i]/a[i,i];
for j:=1 to n do a1[i,j]:=-a[i,j]/a[i,i];  end;
for i:=1 to n do begin  x0[i]:=b1[i];  a1[i,i]:=0;  end;
  2: for i:=1 to n do   begin
  s:=0.0;
  for j:=1 to n do s:=s+a1[i,j]*x0[j];
  x[i]:=b1[i]+s;    end;
         k:=0;
         for i:=1 to n do if abs(x[i]-x0[i])<eps
         then begin k:=k+1; if k=n then goto 1 end
         else begin for j:=1 to n do x0[j]:=x[j]; goto 2 end;
     1:  writeln('Sistemaning taqribiy yechimi:');
       for i:=1 to n do writeln('x[',i:1,']=',x[i]:10:8);
 end.
```

Dastur natijasi



```
CRT - программа завершена
a[1,1]=4
a[1,2]=0.24
a[1,3]=-0.08
b[1]=8
a[2,1]=0.09
a[2,2]=3
a[2,3]=-0.15
b[2]=9
a[3,1]=0.04
a[3,2]=-0.08
a[3,3]=4
b[3]=20
Sistemaning taqribiy yechimi:
x[1]=1.90919900
x[2]=3.19496286
x[3]=5.04480668
```

Chiziqli algebraik tenglamalar sistemasining ildizlarini 0,0000001 aniqlikda iterasiya usulida taqribiy hisoblang.

№1

$$\begin{cases} 5.34X_1 + 0.71X_2 + 0.63X_3 = 2.8 \\ 0.71X_1 - 6.65X_2 - 0.18X_3 = 0.17 \\ 1.17X_1 - 2.35X_2 + 8.75X_3 = 1.28 \end{cases}$$

№2

$$\begin{cases} 3{,}75X_1 - 0{,}28X_2 + 0{,}17X_3 = 0{,}75 \\ 2{,}11X_1 - 5{,}11X_2 - 0{,}12X_3 = 1{,}11 \\ 0{,}22X_1 - 3{,}17X_2 + 11{,}81X_3 = 0{,}05 \end{cases}$$

№3

$$\begin{cases} 8{,}021X_1 - 0{,}18X_2 + 0{,}75X_3 = 0{,}11 \\ 0{,}13X_1 + 7{,}75X_2 - 0{,}11X_3 = 2{,}00 \\ 3{,}01X_1 - 0{,}33X_2 + 10{,}11X_3 = 0{,}13 \end{cases}$$

№4

$$\begin{cases} 9{,}031X_1 - 0{,}28X_2 + 0{,}05X_3 = 0{,}11 \\ 0{,}183X_1 + 7{,}71X_2 - 0{,}19X_3 = 5{,}00 \\ 0{,}01X_1 - 0{,}33X_2 + 3{,}11X_3 = 0{,}19 \end{cases}$$

№5

$$\begin{cases} 3{,}01,\ X_1 - 0{,}14X_2 - 0{,}15X_3 = 1{,}00 \\ 1{,}11X_1 + 8{,}13X_2 - 0{,}75X_3 = 0{,}13 \\ 0{,}17X_1 - 2{,}11X_2 + 4{,}71X_3 = 0{,}17 \end{cases}$$

№6

$$\begin{cases} 2{,}92X_1 - 0{,}83X_2 + 0{,}62X_3 = 2{,}15 \\ 0{,}24X_1 - 2{,}54X_2 + 0{,}43X_3 = 0{,}62 \\ 0{,}73X_1 - 0{,}81X_2 - 4{,}67X_3 = 0{,}88 \end{cases}$$

№7

$$\begin{cases} 5{,}24X_1 - 0{,}87X_2 - 3{,}17X_3 = 0{,}46 \\ 2{,}11X_1 - 6{,}45X_2 + 1{,}44X_3 = 1{,}5 \\ 0{,}48X_1 + 1{,}25X_2 - 3{,}63X_3 = 0{,}35 \end{cases}$$

№8

$$\begin{cases} 7{,}64X_1 - 0{,}83X_2 + 4{,}2X_3 = 2{,}23 \\ 0{,}58X_1 - 2{,}83X_2 - 1{,}43X_3 = 1{,}710 \\ 0{,}86X_1 + 0{,}77X_2 + 2{,}88X_3 = -0{,}54 \end{cases}$$

№9

№10

$$\begin{cases}1{,}32X_1-0{,}42X_2+0{,}85X_3=1{,}32\\0{,}63X_1-1{,}43X_2-0{,}58X_3=-0{,}44\\0{,}84X_1-2{,}23X_2-3{,}52X_3=0{,}62\end{cases}$$

$$\begin{cases}2{,}73X_1-1{,}24X_2-0{,}38X_3=0{,}58\\1{,}25X_1+6{,}66X_2-0{,}78X_3=0{,}66\\0{,}75X_1+1{,}22X_2-7{,}83X_3=0{,}92\end{cases}$$

№11

$$\begin{cases}1{,}62X_1-0{,}44X_2-0{,}86X_3=0{,}68\\0{,}83X_1+1{,}42X_2-0{,}56X_3=1{,}24\\0{,}58X_1-0{,}37X_2-1{,}62X_3=0{,}87\end{cases}$$

№12

$$\begin{cases}1{,}26X_1-0{,}34X_2+0{,}17X_3=3{,}14\\0{,}75X_1+1{,}84X_2-0{,}48X_3=-1{,}17\\0{,}44X_1-1{,}85X_2+11{,}16X_3=1{,}83\end{cases}$$

№13

$$\begin{cases}3{,}46X_1+1{,}72X_2+0{,}53X_3=2{,}44\\1{,}53X_1-5{,}32X_2-1{,}83X_3=2{,}83\\0{,}75X_1+0{,}86X_2+3{,}72X_3=1{,}06\end{cases}$$

№14

$$\begin{cases}2{,}47X_1+0{,}65X_2+0{,}88X_3=1{,}24\\1{,}34X_1+8{,}17X_2+2{,}54X_3=2{,}35\\0{,}86X_1-1{,}73X_2-3{,}08X_3=3{,}15\end{cases}$$

№15

$$\begin{cases}4{,}24X_1+1{,}73X_2-1{,}55X_3=1{,}87\\0{,}34X_1+5{,}27X_2+3{,}15X_3=2{,}16\\3{,}05X_1-1{,}05X_2+6{,}63X_3=-1{,}25\end{cases}$$

№16

$$\begin{cases}2{,}43X_1+1{,}04X_2-0{,}58X_3=2{,}71\\0{,}74X_1+1{,}83X_2+0{,}17X_3=1{,}26\\1{,}43X_1-1{,}58X_2+3{,}83X_3=1{,}03\end{cases}$$

№17

$$\begin{cases}2{,}43X_1+0{,}63X_2+1{,}44X_3=2{,}18\\1{,}64X_1-5{,}83X_2-2{,}45X_3=1{,}84\\0{,}58X_1+0{,}58X_2+3{,}18X_3=0{,}74\end{cases}$$

№18

$$\begin{cases}1{,}94X_1+0{,}62X_2-0{,}95X_3=1{,}43\\2{,}15X_1-4{,}18X_2+0{,}57X_3=2{,}43\\1{,}72X_1-0{,}83X_2+3{,}57X_3=3{,}88\end{cases}$$

№19

$$\begin{cases}4{,}62X_1+0{,}56X_2-0{,}43X_3=1{,}16\\1{,}32X_1-5{,}88X_2+1{,}76X_3=2{,}07\\0{,}73X_1+1{,}42X_2-3{,}34X_3=2{,}38\end{cases}$$

№20

$$\begin{cases}1{,}06X_1+0{,}34X_2+0{,}26X_3=1{,}17\\2{,}54X_1-14{,}16X_2+0{,}55X_3=2{,}53\\1{,}34X_1-0{,}47X_2-3{,}83X_3=3{,}26\end{cases}$$

№21

$$\begin{cases}4{,}21X_1+1{,}13X_2-1{,}45X_3=2{,}87\\0{,}34X_1+8{,}27X_2+3{,}15X_3=2{,}14\\3{,}05X_1-1{,}05X_2+6{,}31X_3=-0{,}25\end{cases}$$

№22

$$\begin{cases}3{,}43X_1+1{,}04X_2-0{,}58X_3=1{,}71\\0{,}74X_1+2{,}83X_2+0{,}17X_3=2{,}26\\1{,}43X_1-1{,}58X_2+9{,}83X_3=4{,}03\end{cases}$$

№23

$$\begin{cases}5{,}43X_1+0{,}63X_2+1{,}44X_3=2{,}08\\1{,}64X_1-4{,}83X_2-2{,}45X_3=1{,}94\\0{,}58X_1+0{,}58X_2+2{,}18X_3=1{,}74\end{cases}$$

№24

$$\begin{cases}2{,}94X_1+1{,}62X_2-0{,}15X_3=1{,}53\\0{,}15X_1-2{,}18X_2+0{,}57X_3=3{,}43\\1{,}72X_1-0{,}83X_2+7{,}57X_3=3{,}18\end{cases}$$

№25

$$\begin{cases}2{,}62X_1+0{,}56X_2-0{,}43X_3=1{,}16\\1{,}32X_1-4{,}88X_2+1{,}76X_3=2{,}97\\1{,}73X_1+1{,}42X_2-4{,}34X_3=5{,}38\end{cases}$$

№26

$$\begin{cases}1{,}96X_1+0{,}34X_2+0{,}26X_3=2{,}17\\2{,}54X_1-5{,}16X_2+0{,}55X_3=2{,}03\\1{,}34X_1+1{,}47X_2-3{,}11X_3=11{,}26\end{cases}$$

Zeydel usuli chiziqli bir qadamli birinchi tartibli ityeratsion usuldir. Bu usul oddiy ityeratsion usuldan shu bilan farq qiladiki, dastlabki yaqinlashish $x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}$ga ko'ra $x_1^{(1)}$ topiladi. So'ngra $x_1^{(1)}, x_2^{(0)}, \ldots, x_n^{(0)}$ ko'ra $x_2^{(1)}$

topiladi va x.k. Barcha $x_1^{(1)}$ lar aniqlangandan so'ng $x_i^{(2)}, x_i^{(3)}, \ldots$lar topiladi. Aniqroq aytganda, hisoblashlar quyidagi sxema bo'yicha olib boriladi:

$$x_1^{(k+1)} = \frac{b_1}{a_{11}} - \sum_{j=2}^{n} \frac{a_{1j}}{a_{11}} x_j^{(k)} \qquad x_2^{(k+1)} = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x_1^{(k+1)} - \sum_{j=3}^{n} \frac{a_{2j}}{a_{22}} x_j^{(k)}$$

$$x_i^{(k+1)} = \frac{b_i}{a_{ii}} = \frac{b_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(k)} \qquad x_n^{(k+1)} = \frac{b_n}{a_{nn}} - \sum_{j=1}^{n-1} x_j^{(k+1)}$$

Ko'pincha Zeydel usuli oddiy ityeratsiya usuliga nisbatan yaxshiroq yaqinlashadi, ammo har doim ham bunday bo'lavyermaydi. Bundan tashqari Zeydel usuli programmalashtirish uchun qulaydir, chunki $x_i^{(k+1)}$ ning qiymati hisoblanayotganda $x_1^{(k)}, \ldots, x_{i-1}^{(k)}$ larning qiymatini saqlab qolishning hojati yo'q.

Misol. Zeydel usuli bilan misolning yyechimi 5 xona aniqlikda topilsin.

$$\begin{cases} 10x_1 + x_2 - 3x_3 - 2x_4 + x_5 = 6 \\ -x_1 + 25x_2 - x_3 - 5x_4 - 2x_5 = 11 \\ 2x_1 + x_2 - 20x_3 + 2x_4 - 3x_5 = -19 \\ x_2 - x_3 + 10x_4 - 5x_5 = 10 \\ x_1 + 2x_2 - x_3 - 2x_4 - 20x_5 = -32 \end{cases}$$

Y e c h i s h . Bu tizimning tenglamalarini mos ravishda 10, 25, - 20, 10, 20 larga bo'lib, quyidagi ko'rinishda yozib olamiz:

$$\begin{cases} x_1 = 0{,}6 - 0{,}1x_2 + 0{,}3x_3 + 0{,}2x_4 - 0{,}1x_5 \\ x_2 = 0{,}44 + 0{,}04x_1 - 0{,}04x_3 + 0{,}2x_4 + 0{,}08x_5 \\ x_3 = 0{,}95 + 0{,}1x_1 + 0{,}05x_2 + 0{,}1x_4 - 0{,}15x_5 \\ x_4 = 1 - 0{,}1x_2 + 0{,}1x_3 + 0{,}5x_5 \\ x_5 = 1{,}6 + 0{,}05x_1 + 0{,}1x_2 + 0{,}05x_3 + 0{,}1x_4 \end{cases}$$

bu yerda $\sum_{\substack{j=1 \\ j \neq i}}^{n} \left| \frac{a_{ij}}{a_{ii}} \right| \le a < 1 \quad (i = 1, 2, \ldots, n)$ shart bajariladi. Haqiqadan ham,

$$\sum_{j=1}^{5} |C_{1j}| = -0{,}1 + 0{,}3 + 0{,}2 - 0{,}1 = 0{,}3 < 1; \qquad \sum_{j=1}^{5} |C_{2j}| = 0{,}28 < 1;$$

$$\sum_{j=1}^{5} |C_{3j}| = 0{,}41 < 1; \qquad \sum_{j=1}^{5} |C_{4j}| = 0{,}5 < 1;$$

$$\sum_{j=1}^{5} |C_{5j}| = 0{,}3 < 1;$$

Dastlabki yaqinlashish $x^{(0)}$ sifatida ozod hadlar ustuni (0,6; 0,44; 0,95; 1; 1,6)
Ityeratsiyaning birinchi qadamini bajaramiz:

$$x_1^{(1)} = 0{,}6 - 0{,}1\, x_2^{(0)} + 0{,}3x_3^{(0)} + 0{,}2x_4^{(0)} - 0{,}1x_5^{(0)} =$$
$$= 0{,}6 - 0{,}1 \cdot 0{,}44 + 0{,}3 \cdot 0{,}95 + 0{,}2 \cdot 1 - 0{,}1 \cdot 1{,}6 = 0{,}881$$
$$x_2^{(1)} = 0{,}44 + 0{,}04\, x_1^{(1)} - 0{,}04x_3^{(0)} + 0{,}2x_4^{(0)} + 0{,}08x_5^{(0)} =$$
$$= 0{,}44 + 0{,}04 \cdot 0{,}881 - 0{,}04 \cdot 0{,}95 + 0{,}2 \cdot 1 - 0{,}08 \cdot 1{,}6 = 0{,}771$$
$$x_3^{(1)} = 0{,}95 + 0{,}1\, x_1^{(1)} + 0{,}05x_2^{(1)} + 0{,}1x_4^{(0)} - 0{,}1x_5^{(0)} =$$
$$= 0{,}95 + 0{,}1 \cdot 0{,}881 + 0{,}05 \cdot 0{,}771 + 0{,}1 \cdot 1 - 0{,}15 \cdot 1{,}6 = 0{,}937$$

$$x_4^{(1)} = 1 – 0,1\ x_2^{(1)} + 0,1x_3^{(1)} +0,5x_5^{(0)} = 1,817$$
$$x_5^{(1)} = 1,6 + 0,05x_1^{(1)} + 0,1x_2^{(1)} + 0,05x_3^{(1)} +0,1x_4^{(1)} = 1,948$$

Keyingi yaqinlashishlarni 6- jadvalda keltiramiz:

6 - jadval

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$x_5^{(k)}$
0	0,6	0,44	0,95	1	1,6
1	0.881	0,771	0,937	1,817	1,948
2	0,973	0,961	0,985	1.974	1,992
3	0,995	0,995	0,999	1,996	1,999
4	0,9995	0,9991	0,9997	1,9995	1,9998
5	0,99992	0,99989	0,99997	1.99991	1,99997
6	0,99999	0,99998	0,99999	1,99999	2.00000

Javob: $x_1 = x_2 = x_3 = 1$; $x_4 = x_5 = 2$

Taqribiy hisoblashlar kompyuter texnologiyasi yordamida oson bajariladi. Buning uchun amaliy dasturlarga yoki dasturlashtirish tillariga murojaat etiladi. Quyida Turbo Paskal dasturlash tilida iteratsiya usuliga tuzilgan dastur matni:

```
uses crt;
type mat=array[1..20,1..20] of real;    vector=array[1..30] of real;
var  ag,temp,a,y,b,z,a2: mat;  temp2:array[1..20,1..20] of integer;
 i,p,q,j,k,n,nn,t: integer;  aa,aas,d,m,x,r,bg,x3,x2: vector;  ii: integer;
 m2,s2,max,l,s,f: real;  h: integer;
 begin   write('Count N=');    readln(n);
  for i:=1 to n do     begin      for j:=1 to n do       begin
       write('a[',i,'][',j,']=');       readln(a[i,j]);      end;
    write('b[',i,']=');      readln(a[i,j+1]);     end;
For i:=1 To n do For j:=1 To n+1 do  a2[i,j]:=a[i,j]/a[i,i];
For i:=1 To n do x[i]:= a2[i,n+1]; repeat
For i:=1 To n do  begin s:=a2[i,n+1]; For j:=1 To n do begin
If j<i Then s:=s-a2[i,j]*x3[j]; If j>i Then s:=s-a2[i,j]*x[j];
end; x2[i]:=x3[i]; x3[i]:=s; end; f:=0; For i:=1 To n do
begin  If Abs(x3[i]-x2[i])>0.00001 Then f:=1; x[i]:=x3[i]; end;
until f<>1;  writeln('Ziydel ildizlari'); for k:=1 to n do
writeln('X[',k,']=', x[k]:5:5); for t:=1 to n do begin
l:=a[t,t]; for j:=1 to n+1 do a[t,j]:=a[t,j]/l; for i:=t+1 to n do begin l:=a[i,t];
for j:=1 to n+1 do a[i,j]:=a[i,j]-a[t,j]*l ; end;
end;
end.
```

Yuqorida berilgan tenglamalar sitemasini Ziydel usulida yechishni ABCPascal dasturi yordamida amalga oshiramiz.

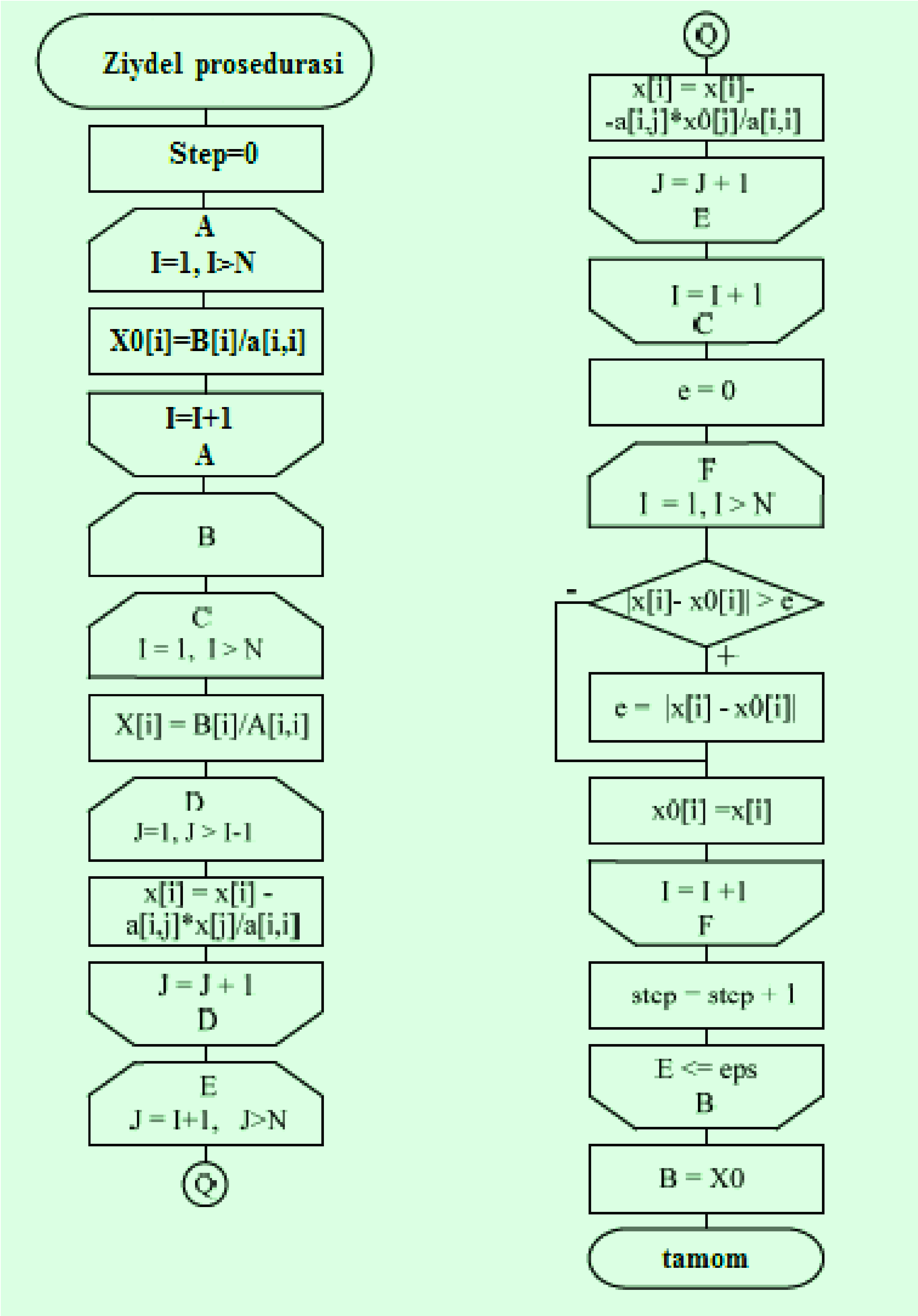
Dastur matni.

```
uses crt;
type mat=array[1..20,1..20] of real; vector=array[1..30] of real;
var  ag,temp,a,y,b,z,a2: mat;  temp2:array[1..20,1..20] of integer;
 i,p,q,j,k,n,nn,t: integer;  aa,aas,d,m,x,r,bg,x3,x2: vector;
 ii: integer;    m2,s2,max,l,s,f: real;    h: integer;
 begin     write('Tenglamalar soni N=');    readln(n);
  for i:=1 to n do     begin
   for j:=1 to n do
      begin  write('a[',i,'][',j,']=');   readln(a[i,j]);   end;
     write('b[',i,']=');    readln(a[i,j+1]);   end;
 For i:=1 To n do
For j:=1 To n+1 do a2[i,j]:=a[i,j]/a[i,i];
For i:=1 To n do x[i]:= a2[i,n+1];
repeat
For i:=1 To n do   begin   s:=a2[i,n+1];
For j:=1 To n do  begin
If j<i Then s:=s-a2[i,j]*x3[j]; If j>i Then s:=s-a2[i,j]*x[j]; end;
x2[i]:=x3[i]; x3[i]:=s;  end;  f:=0;
For i:=1 To n do    begin  If Abs(x3[i]-x2[i])>0.00001 Then f:=1;
x[i]:=x3[i];  end;
until f<>1;
writeln('Ziydel ildizlari'); for k:=1 to n do
writeln('X[',k,']=', x[k]:5:5);
for t:=1 to n do   begin   l:=a[t,t];  for j:=1 to n+1 do
a[t,j]:=a[t,j]/l;
for i:=t+1 to n do  begin
l:=a[i,t];
for j:=1 to n+1 do a[i,j]:=a[i,j]-a[t,j]*l;
 end;    end;  end.
```

Dastur natijasi :

```
CRT - программа завершена
a[3][1]=2
a[3][2]=1
a[3][3]=-20
a[3][4]=2
a[3][5]=-3
b[3]=-19
a[4][1]=0
a[4][2]=1
a[4][3]=-1
a[4][4]=10
a[4][5]=-5
b[4]=10
a[5][1]=1
a[5][2]=2
a[5][3]=-1
a[5][4]=-2
a[5][5]=-20
b[5]=-32
Ziydel ildizlari
X[1]=1.01625
X[2]=0.99756
X[3]=1.04988
X[4]=1.76597
X[5]=1.52148
```

Ziydel usuliga tuzilgan algoritm blok – sxemasi:



Quyidagi tenglamalar sistemasini Zeydel usulida ε=0,000001 aniqlikda yeching. (N-talabaning tartib raqami)

$$\begin{cases} 12{,}1x_1-0{,}87x_2+4{,}1x_4+x_5-2{,}8x_6+1{,}1x_7-x_9+0{,}24x_{10}=N-1 \\ 0{,}8x_1-19{,}8x_2+0{,}7x_3-1{,}1x_4+0{,}7x_5-0{,}92x_6+x_7-1.5x_8-2x_9+1{,}35x_{10}=3-N \\ 1{,}2x_1+1{,}5x_2+20{,}7x_3-1.9x_4-0{,}6x_5-0{,}22x_6+0.5x_8-1{,}4x_9+2{,}75x_{10}=N+2 \\ 1{,}43x_1-1{,}18x_2+0{,}2x_3-22x_4-1{,}27x_5-0{,}66x_6-x_7+1{,}15x_8-x_9+1{,}4x_{10}=-5{,}1 \\ 3{,}1x_1+1{,}71x_3-1{,}02x_4+33{,}1x_5-1{,}85x_6-3x_7-1{,}9x_8-4{,}5x_9+0{,}16x_{10}=8{,}97 \\ 0{,}42x_1-1{,}4x_2-0{,}17x_3-1{,}83x_4+3{,}6x_5-27{,}92x_6-0{,}8x_7+2.5x_8-x_9+5x_{10}=-9{,}88 \\ -1{,}44x_2+3{,}7x_3-x_4+x_5-0{,}99x_6+18{,}78x_7+5{,}5x_8+0{,}2x_9+0{,}75x_{10}=10 \\ 7{,}8x_1-10{,}8x_2+0{,}1x_3+4{,}1x_4+0{,}3x_5-0{,}72x_6+4{,}8x_7-39{,}5x_8-0{,}87x_9+3x_{10}=-N \\ 5{,}2x_1+1{,}01x_2+0{,}42x_3-x_4+1{,}17x_5+0{,}2x_6+2x_7-1{,}5x_8+19{,}2x_9+0{,}63x_{10}=63{,}3 \\ 0{,}09x_1-1{,}26x_2+0{,}15x_3-1{,}01x_4+0{,}19x_5-2{,}52x_6+0{,}47x_7-0{,}41x_8+0{,}2x_9+19{,}5x_{10}=20 \end{cases}$$

Nazorat savollari

1. Algebraik va transendent tenglamalar sistemalarini yechishning qanday usullarini bilasiz.
2. Algebraik va transendent tenglamalar sistemalarini yechishning Gauss usuli nima ?
3. Algebraik va transendent tenglamalar sistemalarini yechishning Gauss usuliga algoritm tuzing?
4. Algebraik va transendent tenglamalar sistemalarini yechishda qaysi holatda Gauss usulini qo’llash mumkin?
5. Iterasiya usullari haqida umumiy tushuncha bering?
6. Iterasion jarayon yaqinlashish shartini tushuuntirib bering?
7. Oddiy iterasiya usuli algortmi haqida tushuntiring?
8. Chiziqli tenglamalar tizimini yechish usullari haqida ma’lumot bering.
9. Zeydel usulini qo’llab chiziqli tenglamalar tizimini yechish algoritmini tuzing.
10. Zeydel usulini qo’llab chiziqli tenglamalar tizimini yechish dasturini tuzing.

8-AMALIY ISH

Mavzu: Empirik formulalar. Funksiyalarni eng kichik kvadratlar usuli bilan approksimatsiyalash va uning algoritmi

Ishning maqsadi: Tajriba natijalarini eng kichik kvadratlar usuli bilan approksimatsiyalash bo'yicha amliy ko'nikmani shakllantirish

Nazariy qism

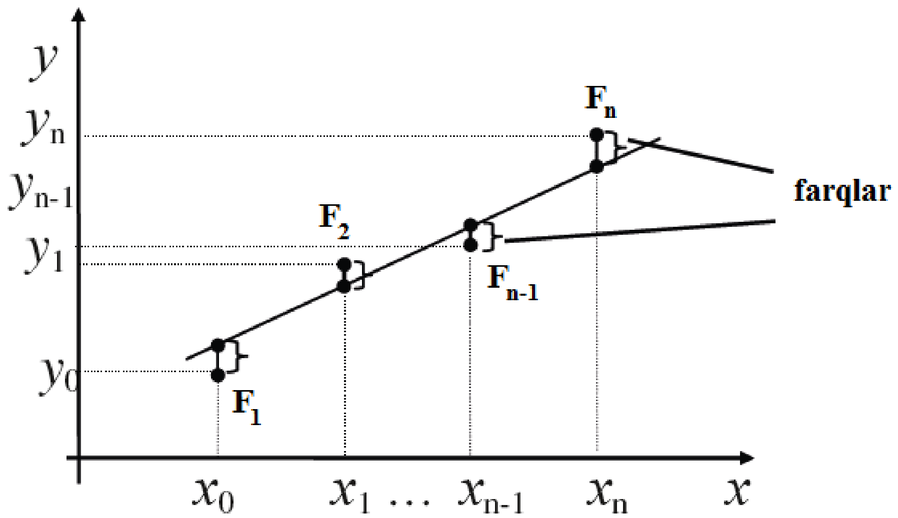
Tajriba natijalarini q'ayta ishlash. Eslatib o'tamiz, interpolyatsiya deganda erkli o'zgaruvchining diskret nuqtalari bilan funksiyaning shu nuqtalardagi mos q'iymatlari orasidagi munosabati ma'lum bo'lgan holda funksional bog'lanishning taqribiy yoki aniq analitik ifodasini tuzish tushuniladi. Ko'pincha kuzatishlar va tajribalar orqali empirik formulalarni keltirib chiq'arish mumkin. Masalan, haroratning ko'tarilishi yoki aksincha pasayishini, simob ustunining ko'tarilishi yoki pasayishiga qarab bilish mumkin. Demak, harorat bilan simob ustini o'rtasidagi chiziqli bog'lanish borligini tajriba orqali bilish mumkin. Bunday masalalarni yechishda **eng kichik kvadratlar** usulidan foydalanamiz.

Emperik bog'liqlikni aniqlash. Qandaydir funksiyaning qiymatlari jadval ko'rinishida berilgan bo'lsin. U holda bu funksiyani jadval funksiya ham deb ataymiz.

X	X_1	X_2	…	X_n
Y	Y_1	Y_2	…	Y_n

Bu yrda berilgan tajriba natijalarini bog'lovchi emperik funksiya sifatida ushbu $y = f(x)$ funksiyani aniqlash masalasini ko'rib chiqamiz.

Jadvaldagi qiymatlar bo'yicha $F(x_i, y_i)$ nuqtalarni Dekart koordinatalar sistemasida tasvirlaymiz.



$y = f(x)$ funksiya uchun $y_i \approx f(x_i)$ shart o'rinli bo'lsin. Bu yerda shart xatoligi $\varepsilon_i = y_i^0 - y_i$ qabul qilinsin. Bu yerda $y_i^0 = f(x_i)$.

$\varphi_0(x), \varphi_1(x), \varphi_2(x),..., \varphi_m(x)$ - bazis funksiyalar bo'lsin va bu funksiyalar yordamida $y = \Phi_m(x) \equiv c_0\varphi_0(x) + c_1\varphi_1(x) + ... + c_m\varphi_m(x)$ (7.1) funksiyani hosil qilamiz va bu yerda $\Phi_m(x)$ ko'phadga ega bo'lamiz.

(7.1) – formulada c_j $(j = 1,...,m)$ koeffitsiyentlarni aniqlashda eng kichik kvadratlar usulidan foydalanamiz ya'ni,

$$\delta_m = \sum_{i=0}^{n}\left(y_i - \Phi_m(x_i)\right)^2 = \sum_{i=0}^{n}\left(y_i - c_0\varphi_0(x_i) - ... - c_m\varphi_m(x_i)\right)^2 \quad (7.2)$$

funksiyaning minimumini topamiz. Demak, shunday $c_j \ (j = 1,...,m)$ noma'lumlarni aniqlash lozimki natijada δ_m funksiyaning qiymati eng kichik bo'lsin. Ma'lumki, ihtiyoriy x uchun $\delta_m \geq 0$ shart bajariladi. δ_m funksiyaning $c_j \ (j = 1,...,m)$ argumentlaridagi birinchi tartibli hususiy hosilalarini hisoblaymiz va ularni nolga tenglaymiz. Natijada quyidagi tenglamalar sistemasiga ega bo'lamiz.

$$\begin{cases} \dfrac{\partial \delta_m}{\partial c_0} = -2\sum_{i=0}^{n}(y_i - c_0\varphi_0(x_i) - ... - c_m\varphi_m(x_i))\cdot\varphi_0(x_i) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{\partial \delta_m}{\partial c_m} = -2\sum_{i=0}^{n}(y_i - c_0\varphi_0(x_i) - ... - c_m\varphi_m(x_i))\cdot\varphi_m(x_i) = 0 \end{cases} \quad (7.3)$$

Agar $(f,g) = \sum_{i=0}^{n} f(x_i)\cdot g(x_i)$ tenglikdan foydalansak uholda (7.3) –sistemani quyidagicha yozish mumkin bo'ladi.

$$\begin{cases} c_0(\varphi_0,\varphi_0) + c_1(\varphi_0,\varphi_1) + ... + c_m(\varphi_0,\varphi_m) = (\varphi_0, y) \\ c_0(\varphi_1,\varphi_0) + c_1(\varphi_1,\varphi_1) + ... + c_m(\varphi_1,\varphi_m) = (\varphi_1, y) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ c_0(\varphi_m,\varphi_0) + c_1(\varphi_m,\varphi_1) + ... + c_m(\varphi_m,\varphi_m) = (\varphi_m, y) \end{cases} \quad (7.4)$$

$\varphi_k(x) = x^k$ ekanligidan (7.4) sistemani quyidagicha yozish mumkin.

$$\sum_{j=0}^{m}\left(\sum_{j=0}^{n} x_i^{j+k}\right)c_j = \sum_{i=0}^{n} y_i x_i^k, \quad (k = 0,1,...,m) \qquad (7.5)$$

Endi m=1 va m=2 uchun (7.5) sistemani aniqlaymiz.

m=1 bo'lsin. U holda $P_1(x) = c_0 + c_1x$ chiziqli ko'phadga ega bo'lamiz va c_0, c_1 larni aniqlash uchun (7.5) sistemaning ko'rinishi quyidagicha bo'ladi.

$$\begin{cases} (n+1)\,c_0 + (\sum_{i=0}^{n} x_i)\,c_1 = \sum_{i=0}^{n} y_i \\ (\sum_{i=0}^{n} x_i)\,c_0 + (\sum_{i=0}^{n} x_i^2)\,c_1 = \sum_{i=0}^{n} y_i x_i. \end{cases}$$

m=2 bo'lsin. U holda $P_1(x) = c_0 + c_1x + c_2x^2$ kvadrat uchhadga ega bo'lamiz va c_0, c_1, c_2 larni aniqlash uchun (7.5) sistemaning ko'rinishi quyidagicha bo'ladi.

$$\begin{cases} (n+1)c_0 + (\sum_{i=0}^{n} x_i)c_1 + (\sum_{i=0}^{n} x_i^2)c_2 = \sum_{i=0}^{n} y_i \\ (\sum_{i=0}^{n} x_i)c_0 + (\sum_{i=0}^{n} x_i^2)c_1 + (\sum_{i=0}^{n} x_i^3)c_2 = \sum_{i=0}^{n} y_i x_i \\ (\sum_{i=0}^{n} x_i^2)c_0 + (\sum_{i=0}^{n} x_i^3)c_1 + (\sum_{i=0}^{n} x_i^4)c_2 = \sum_{i=0}^{n} y_i x_i^2 \end{cases}$$

Misol-1. Berilgan jadval qiymatlari asosida tajriba natijalarini eng kichik kvadratlar usuli bilan approksimatsiyalang (T=4,2).

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	5	4	8	1	11	9	2	0	7	4

Yechish. m=1 bo'lsin. U holda quyidagi jadvalni MS Excel dasturi yordamida hisoblaymiz.

N	X	Y	XY	X2
1	0	5	0	0
2	1	4	4	1
3	1,5	8	12	2,25
4	3	1	3	9
5	4	11	44	16
6	5	9	45	25
7	8	2	16	64
8	8,5	0	0	72,25
9	9	7	63	81
10	9,5	4	38	90,25
	49,5	51	225	360,75

$P_1(x) = c_0 + c_1 x$ regressiyani noma'lum koeffitsentlarini hisoblaymiz. Buning uchun tenglamalar sitemasini tuzamiz (n=9).

$$\begin{cases} 10c_0 + 49{,}5c_1 = 51 \\ 49{,}5c_0 + 360{,}75c_1 = 225 \end{cases}$$

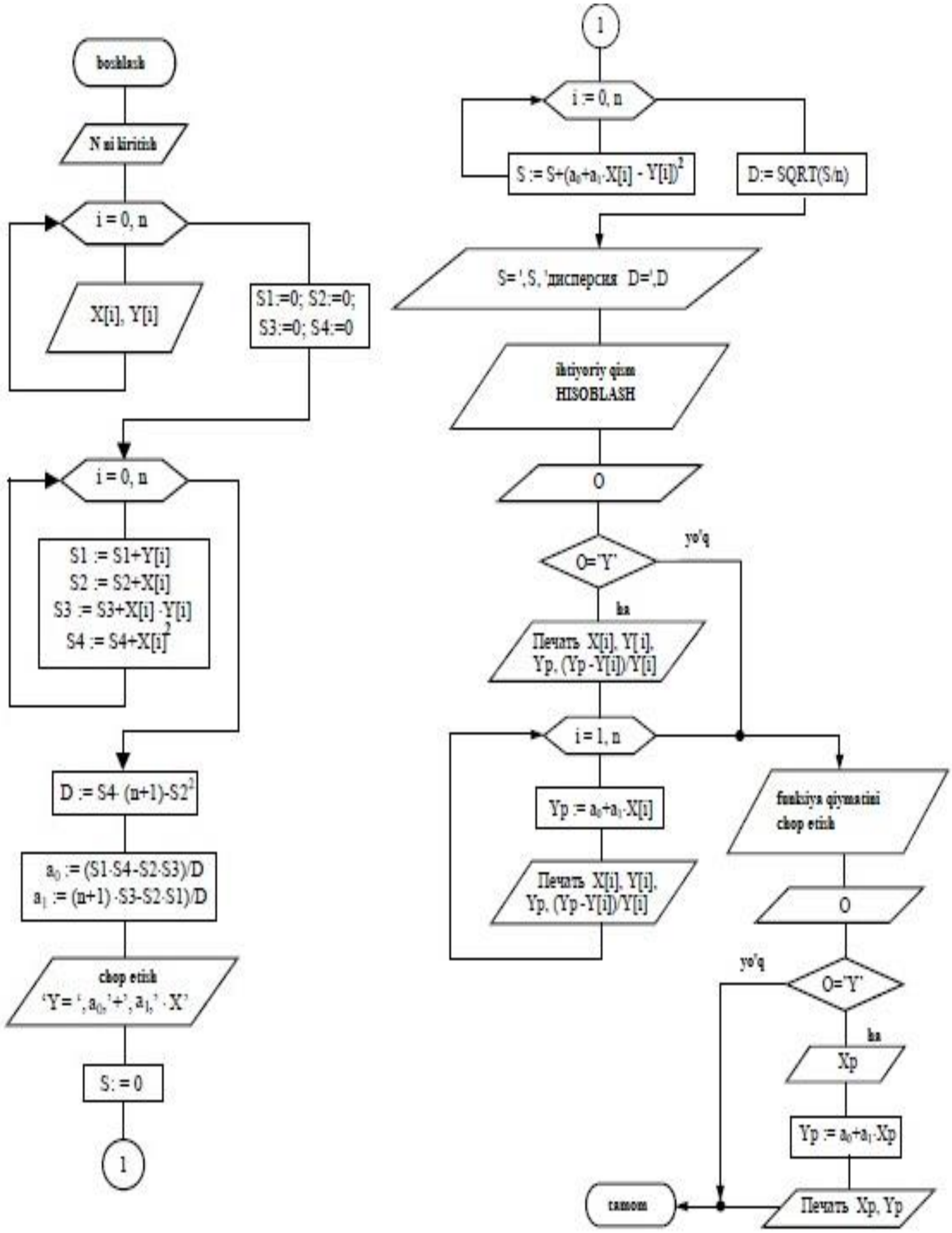
Bu sistemani yechib $c_0 = 6{,}274$, $c_1 = -0{,}237$ ildizlarga ega bo'lamiz. U holda $P_1(x)$ regressiya ko'phadining ko'rinishi quyidagicha bo'ladi: $P_1(x) = 6{,}274 - 0{,}27x$

Ushbu jadvalga ega bo'lamiz:

Y	P(X)
5	6,2741413
4	6,036941
8	5,9183409
1	5,5625405
11	5,3253402
9	5,08814
2	4,3765392
0	4,2579391
7	4,139339
4	4,0207388
51	51

$y(4{,}2) = 6{,}274 - 0{,}27 \cdot 4{,}2 = 5{,}278$

Eng kichik kvadratlar usuliga tuzilgan algoritm blok-sxemasi



1-misolda keltirilgan tajriba natijalarini qayta ishlash uchun eng kichik kvadratlar usuliga tuzilgan ABC Pascal dasturlash tilidagi dastur matni keltirilgan. Bu yerda $T = x_0 = 4{,}2$ deb qaralsin.

```
Program Kvadrat; uses crt;
const n=10;
var x0,y0,a,b,c1,c2,p1,p2:real;
i:integer;
x,y:array[1..n] of real;
begin
 writeln('Qaysi qiymat uchun hisoblaymiz');
write('x0='); readln(x0);
writeln('Massiv elementlarini kiriting');
for i:=1 to n do begin
write('x',I:1,'=');readln(x[I]);
write('y',I:1,'=');readln(y[I]); end;
c1:=0; c2:=0; p1:=0; p2:=0;
for I:=1 to n do begin
c1:=c1+x[I]*x[I];
c2:=c2+x[I];
p1:=p1+x[I]*y[I];
p2:=p2+y[I];
 end;
a:=(p1*n-p2*c2)/(c1*n-c2*c2);
b:=(p2-a*c2)/n;
writeln('a=',a);writeln('b=',b);
y0:=a*x0+b;
writeln('y0=',y0:5:6);
begin
   for I:=1 to n do begin
         y0:=a*x[i]+b;
         writeln('y',I:1,'=',y0:5:6);end;end;
end.
```

Dastur natijasi

```
CRT - программа завершена
y5=11
x6=5
y6=9
x7=8
y7=2
x8=8.5
y8=0
x9=9
y9=7
x10=9.5
y10=4
a=-0.237200259235256
b=6.27414128321452
y0=5.277900
y1=6.274141
y2=6.036941
y3=5.918341
y4=5.562541
y5=5.325340
y6=5.088140
y7=4.376539
y8=4.257939
y9=4.139339
y10=4.020739
_
```

Mustaqil yechish uchun misollar

Berilgan jadval qiymatlari asosida tajriba natijalarini eng kichik kvadratlar usuli bilan approksimatsiyalang (chiziqli regressiya T=5,3).

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	1	1,12	1,87	2,25	5,01	5,27	7,14	9,21	8,42	10,9

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	2	3	4	5	6,5	7,5	8	8,5
Y	-0,4	1,8	3,7	2,8	4,1	4,8	7,2	8,01	7,57	9,21

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	1	0,7	2	2,4	4,5	4,8	7,77	9	8,8	10

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0	1,5	2	2,5	4,2	5,5	8,5	7,8	8,9	11,5

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0.5	1,5	2,5	2,9	3,7	5,1	7,8	7	8	10,2

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0,9	1,1	1,5	2,9	3	4,1	8,5	8,9	8	9,4

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0,8	1,2	1,5	2,2	3,8	4,9	8,8	9,9	8,5	9

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0,1	1,9	1,6	2,2	4,5	4,5	8,8	9,4	10	8,7

9-AMALIY ISH

Mavzu: Chiziqli programmalash masalasini yechishning simpleks usuli va uning algoritmi. Transport masalasi

Ishning maqsadi: Chiziqli programmalash masalasini yechish bo'yicha amaliy ko'nikmalarni hosil qilish

Nazariy qism.

Chiziqli dasturlashtirish masalasining berilishi. Quyidagi formula bilan berilgan cheklanishlar tizimini, ya'ni

$$\sum_{i=1,n;\, j=1,m} a_{ij}x_j \geq b_i (\leq b_i) \qquad (8.1)$$

qanoatlantiruvchi nuqtalar orasida ushbu

$$F(x_j) = \sum c_j x_j \qquad (8.2)$$

maqsad funksiyaga eng katta (eng kichik) qiymat beruivchisini topish talab etilsin. Bu masala dasturlashtirish masalasi deyiladi.

(8.1) – cheklanishlar tizimi tenglama, tengsizliklar yoki tenglama va tengsizliklardan iborat sistemadan iborat bo'lishi mumkin.

Agar (8.2) – formula chiziqli bo'lsa programmalashtirish masalasi chiziqli programmalashtirish masalasi deyiladi.

Chiziqli programmalashtirish masalasini amaliy dasturlarda yechish ancha qulay hisoblanadi. MS Excel dasturida «Поиск решения» ustqurmasi yoki MatLab tizimida "Optimallashtirish" ilovasi yordamida hisoblashlarni misol keltirish mumkin. Shuningdek MathCad tizimida ham chiziqli programmalashtirish masalasini osonlik bilan yechish mumkin.

MathCad da chiziqli programmalashtirish masalasini yechishda ***maximize*** va ***minimize*** funktsiyalaridan foydalanish mumkin. Bu funktsiyalar umumiy holda quyidagi ko`rinishda yoziladi:

Maximize(F,<o`zgaruvchilar ro`yxati>)

Minimize(F,<o`zgaruvchilar ro`yxati>)

MathCad da chiziqli programmalashtirish masalasini yechish quyidagicha bajariladi:

1.MathCad ni ishga tushurgandan so`ng, maqsad funktsiyasi yoziladi, masalan f(x,y)=<funktsiya ko`rinishi> va o`zgaruvchilarning boshlang`ich qiymati kiritiladi.

2.Given kalit so`zi yoziladi.

3.Tengsizliklar tizimi va cheklanishlar kiritiladi.

4.Biror o`zgaruvchiga maximize yoki minimize funktsiyasi yuboriladi.

5.Shu o`zgaruvchi yozilib tenglik kiritiladi. Natija vektor ko`rinishida hosil bo`ladi.

6.Maqsad funktsiyasi qiymatini hisoblash uchun, masalan f (p_0,p_1) yozilib tenglik belgisi kiritiladi.

1-vazifa. Berilgan chiziqli dasturlash masalalsini MathCad dasturidan foydalanib yeching.

Masala. Sanoat korxonasida sozlash – ta'mirlash ishlarini olib borish uchun ikki uskuna uchun to'rt turdagi materiallar ishlatiladi. 1-tur materialdan A1 birlik, 2-tur materialdan A2 birlik, 3-tur materialdan A3 birlik, 4-tur materialdan A4 birlik miqdorda mavjud. Birinchi uskuna uchun 1-tur materialdan 5 birlik, 2-tur materialdan 7 birlik, 3-tur materialdan 6 birlik, 4-tur materialdan 6 birlik sarflansa bu kattalik ikkinchi uskuna uchun mos ravishda 4 birlik, 8 birlik, 3 birlik va 9 birliklarni tashkil etadi. 200000 so'm 1 dona 1-uskunadan, 150000 so'm 1 dona 2-uskunadan tejaladigan foyda ma'lum bo'lsa, tamirlash taqsimotining optimal rejasini aniqlang.

Masalaning matematik modeli:

x_1 - ta'mirlanayotgan 1-uskunalar soni, x_2 - ta'mirlanayotgan 2-uskunalar soni bo'lsin. U holda quyidagi cheklanishlar tizimi o'rinli bo'ladi. Nomanfiy o'zgaruvchilar uchun:

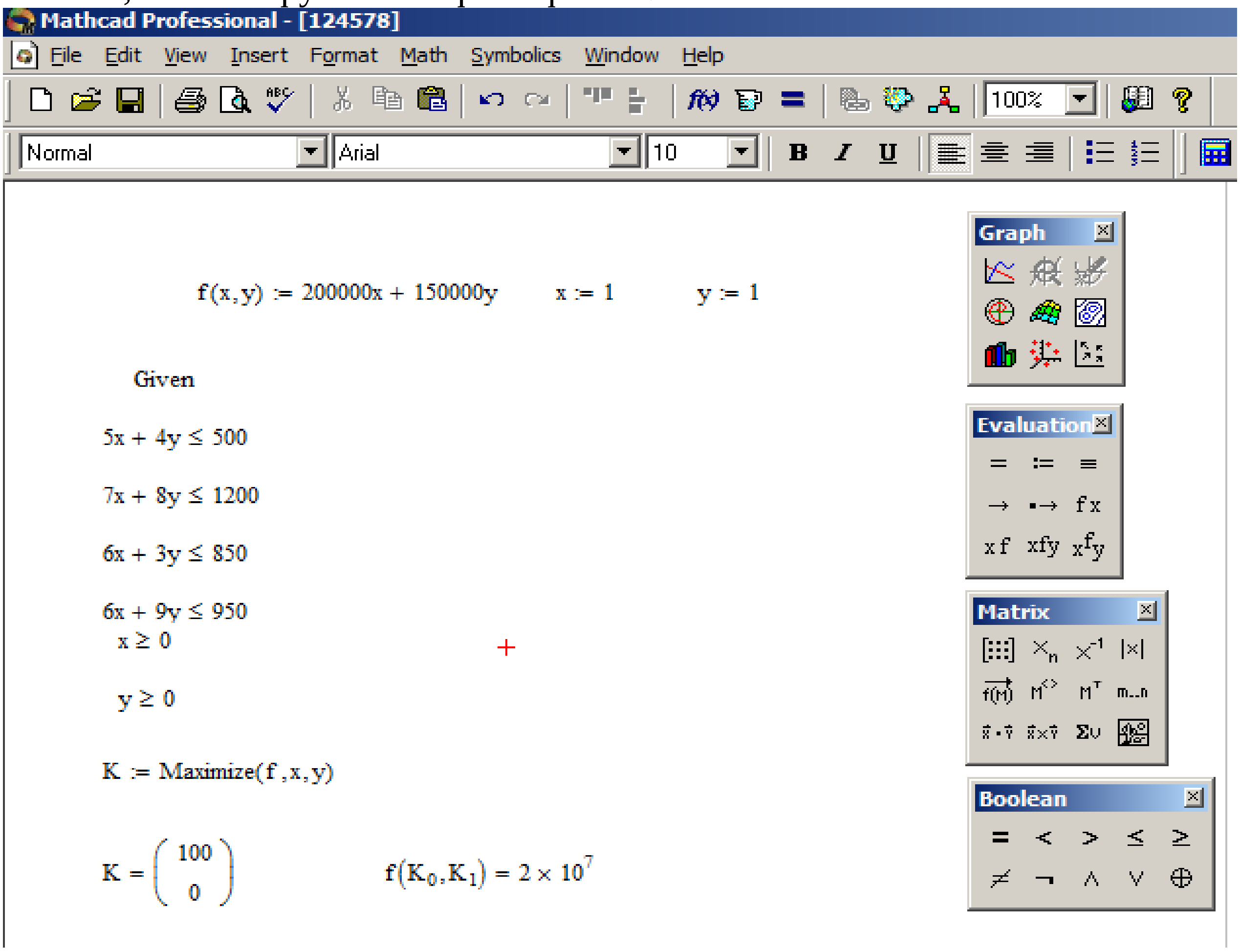
$$\begin{cases} 5x_1 + 4x_2 \le A_1 \\ 7x_1 + 8x_2 \le A_2 \\ 6x_1 + 3x_2 \le A_3 \\ 6x_1 + 9x_2 \le A_4 \end{cases}$$

Optimal reja natijasiga erishtiruvchi maqsad funksiya ko'rinishi esa quyidagicha bo'ladi.

$$F(x_1, x_2) = 200000\,x_1 + 150000\,x_2$$

Demak shartga ko'ra $F(x_1, x_2) \to \max$ o'rinli bo'ladi.

Masalani MathCad dasturidan foydalanib hisoblaymiz: A1=500, A2=1200, A3=850, A4=950 qiymatlarni qabul qilamiz.



NATIJA:

Faqat 1-uskunalardan 100 donasini ta'mirlash kerak.

Amaliy ish topshirig'i.

Berilgan maqsad funksiya uchun minimal qiymatini hisoblang

$$F(x, y) = N^2 x + (N-2)(N+1)y \rightarrow \min$$

Bu yerda N=100-k (k=1,2,3,...,28)

Amaliy ish variantlari

Materiallar	**A1**	**A2**	**A3**	**A4**
variantlar				
1	120	480	605	450
2	210	490	600	500
3	281	500	595	550
4	245	510	590	600
5	150	520	585	410
6	140	530	580	510
7	190	540	575	610
8	320	550	570	402
9	140	560	565	502
10	158	570	560	602
11	145	580	555	460
12	169	590	550	560
13	224	600	545	660
14	280	610	540	413
15	189	620	535	513
16	174	630	530	613
17	125	640	525	475
18	212	650	520	575
19	201	670	515	675
20	200	680	510	435
21	150	690	505	535
22	160	700	500	635
23	170	710	495	420
24	122	720	490	520
25	129	730	485	620
26	127	740	480	492
27	210	750	475	592
28	240	760	470	692

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Qarshi-2019