O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

QARSHI MUHANDISLIK-IQTISODIYOT INSTITUTI
ELEKTRONIKA VA AVTOMATIKA fakulteti

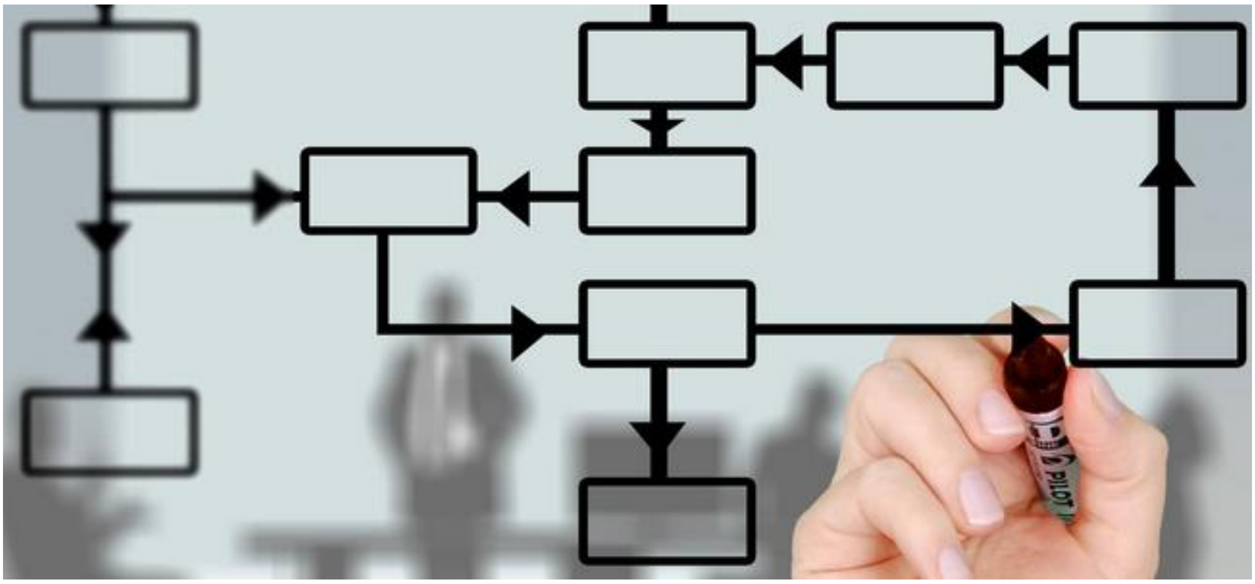
“TEXNOLOGIK JARAYONLARNI AVTOMATLASHTIRISH VA BOSHQARUV” kafedrasi

“HISOBLASH USULLARINI ALGORITMLASH” FANIDAN

LABORATORIYA MASHG’ULOTLARNI

bajarish bo’yicha

USLUBIY QO’LLANMA



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Uslubiy qo’llanma 5311000-Texnologik jarayonlar va ishlab chiqarishni avtomatlashtirish va boshqarish (kimyo, neft-kimyo va oziq-ovqat sanoati)” ta'lim yo‘nalishi talabalari uchun “Hisoblash usullarini algoritmlash” fanidan laboratoriya mashg’ulotlarini bajarishda foydalanishga mo’ljallangan.

Uslubiy qo’llanma “Texnologik jarayonlarni avtomatlashtirish va boshqarish” kafedrasi yig’ilishida (№_______qaror,________), Elekronika va avtomatika fakulteti Uslubiy kengashida (№_______qaror,_______), QMII Uslubiy kengashida (№ _______qaror,______) ko’rib chiqildi va chop etishga tavsiya etildi.

KIRISH

Ishlab chiqarish texnika sohasida zamonaviy loyihalar, ilmiy tadqiqot ishlari jarayonida muhandislik hisoblashlari uning ajralmas muhim qismi hisoblanadi. Bunda mutaxassislar tomonidan amalgam oshiriladigan hisoblash ishlari «Hisoblash usullarini algoritmlash» fani tushunchalari bilan bevosita bog’liqdir. Shu sababli ushbu fan dasturi asosida zamonaviy o’quv adabiyotlarini yaratish dolzarb masalalardan biri sanaladi. Obektlar, hodisalar va jarayonlarni matematik modellashtirish asosida matematik modellarni kompyuterda amalga oshirish, ya’ni matematik modellar tenglamalarini sonli usullar bilan kompyuterda taqribiy yechish algoritmlarini ishlab chiqish, algoritmlarni amalga oshiruvchi dasturiy ta’minot yaratish, kompyuterda hisoblash tajribalarini o‘tkazish, tajriba natijalarini tahlil qilish, xulosa chiqarish va qaror qabul qilish yotadi.

“Hisoblash usullarini algoritmlash” fani buyicha tayyorlangan mazkur qo’llanma fanning o’quv dasturi asosida tayyorlangan. Ta’lim yo’nalishi malaka talablari hisobga olingan.

Ushbu uslibiy qo’llanmada mavzularga oid amaliy masalalar yechilish uslublari bilan ularga tuzilgan algoritmlarni, hamda hisoblash natijalarini yoritadi. Shuningdek amaliy matematikada hozirgi kunda keng qo’llanilib kelinayotgan amaliy dasturlar va dasturlash texnologiyasi yushunchalari ham keltirilgan.

Uslubiy qo’llanma 5311000-“Texnologik jarayonlar va ishlab chiqarishni avtomatlashtirish va boshqarish (kimyo, neft – kimyo va oziq – ovqat sanoati)” ta’lim yo’nalishi talabalari va hisoblash matematikasi bilan qiziquvchi ommaga mo’ljallangan.

1-Laboratoriya ishi

Algebraik va transendent tenglamalarni yechishning oddiy iteratsiya va kesmani teng ikkiga bo‘lish usullari va ularning algoritmi.

Ishdan maqsad: Algebraik va transendent tenglamalarni yechimini to’gri va iterasion usullar bilan olishni o’rganish.

Nazariy qism

Amaliyotda ko’pincha

$$f(x)=0 \qquad (1.1)$$

kabi tenglamalarning ildizini taqribiy hisoblab topishga to’g’ri keladi.

1.1-teorema. Aytaylik,

1) f(x) funktsiya [a,b] kesmada uzluksiz va (a,b) intervalda hosilaga ega bo‘lsin;
2) $f(a)\cdot f(b)<0$, ya’ni f(x) funktsiya kesmaning chetlarida har xil ishoraga ega bo‘lsin;
3) f’(x) hosila (a,b) intervalda o‘z ishorasini saqlasin.

U holda, (1.1) tenglama [a,b] oraliqda yagona yechimga ega bo‘ladi.

$f(x)=0$ tenglama berilgan bo‘lsin. $[a,b]$ kesmada $u=f(x)$ funktsiya 1.1-teoremaning barcha shartlarini qanoatlantirsin.

<u>Oraliqni teng ikkiga bo‘lish usuli.</u> $[a,b]$ oraliqni $x_0=(a+b)/2$ nuqta orqali ikkita teng $[a,x_0]$ va $[x_0,b]$ oraliqlarga ajratamiz. Agar $|a-x_0|\leq\varepsilon$ bo‘lsa, $x=x_0$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi. Bu shart bajarilmasa, $[a,x_0]$ va $[x_0,b]$ oraliqlardan (1) tenglama ildizi joylashganini tanlab olamiz va uni $[a_1,b_1]$ deb belgilaymiz. $x_1=(a_1+b_1)/2$ nuqta yordamida $[a_1,b_1]$ oraliqni ikkita teng $[a_1,x_1]$ va $[x_1,b_1]$ oraliqlarga ajratamiz. $|a_1-x_1|\leq\varepsilon$ bo‘lsa, $x=x_1$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi, aks holda $[a_1,x_1]$ va $[x_1,b_1]$ oraliqlardan (1) tenglama ildizi joylashganini tanlab olamiz va uni $[a_2,b_2]$ deb belgilaymiz. Bu oraliq uchun yuqoridagi hisoblashlar ketma-ketligini $|a_i-x_i|\leq\varepsilon$ (i=2,3,4,…) shart bajarilguncha davom ettiramiz. Natijada (1) tenglamaning $x=x_i$ taqribiy yechimini hosil qilamiz.

1.1-masala. $e^x-10x-2=0$ tenglama yechimi kesmani teng ikkiga bo‘lish usulida $\varepsilon=0{,}01$ aniqlik bilan toping.

Yechish. $f(x)=e^x-10x-2$ funksiyaning 1.1-teoremaning barcha shartlarini qanoatlantiradigan aniqlanish sohasini topish lozim.Agar bu oraliq mavjud bo’lsa tenglamaga kesmani teng ikkiga bo‘lish usulini ishlatish mumkin.

Tenglama ildizini ajratish dasturini keltiramiz. Dastur natijasiga ko’ra [-0,2;-0,1] kesmani yoki [-1;0] kesmani tanlashimiz mumkin.

$e^x-10x-2=0$ tenglama ildizini ajratish dastur matni quyidagicha bo’ladi:

```
program ildizni_ajratish; uses crt;
label 1,2;
     var a,c,x,fa,fc,h:real;
             i,M,N:integer;
        function f(x:real):real;
        begin
     f:=exp(x)-10*x-2;
        end;
begin clrscr;
     2: write('a='); read(a);
      write('N='); read(N);
       write('M='); read(M);
      h:=1/N;

begin
for i:=1 to M do begin c:=a+h; fa:=f(a);fc:=f(c);
if fa*fc<0 then goto 1;
a:=c;end; end; goto 2;
 1: writeln('a=',a:5:3);
  writeln('b=', c:5:3);
    end.
```

Dastur natijasi

```
CRT - программа завершена
a=-1
N=10
M=100
a=-0.200
b=-0.100
_
```

1) [-1,0] oraliqni t_0=(-1+0)/2=-0.5 nuqta yordamida teng ikkiga bo‘lamiz. $f(t_0)=e^{-0,5}+5-2>0$, $f(-1)=8,386>0$, $f(0)=-1<0$ bo‘lganligi uchun yechim [-0.5, 0] oraliqda yotadi.

2) bu oraliqni t_1=(-0,5+0)/2=-0,25 nuqta yordamida teng ikkiga bo‘lamiz. $f(-1)\cdot f(-0,25)=8,386\cdot 1,279>0$ bo‘lganligi uchun yechim [a_2; b_2]=[-0,25;0] oraliqda yotadi.

Aniqlik $|b_2-a_2|=0,25>2e$ etarli bo‘lmagani uchun [-0,25;0] oraliqni $t_2=\dfrac{0-0,25}{2}=0,125$ nuqta yordamida teng ikkiga bo‘lamiz.

3) *f*(-0.125)=0.132 >0 bo‘lganligi uchun yechim [a_3,b_3]= [-0,12; 0] oraliqda yotadi. Aniqlik |a_3-b_3|=0,125>2e=0,02 etarli bo‘lmagani uchun [-0.125,0] oraliqni $t_3=\dfrac{0-0,125}{2}=0,063$ nuqta yordamida teng ikkiga bo‘lamiz.

4). $f(-0,063)=-0,461$ va $f(-0,125)=0,132$ bo‘lgani uchun yechim [a_4,b_4] =

[-0,125; -0,063] oraliqda yotadi. $|a_4 - b_4|$=0,062>2e=0,02 etarli bo‘lmaganligi uchun [-1,12; -0,063] oraliqni t_4= (- 0,125 – 0, 063)/2= - 0,094 nuqta yordamida teng ikkiga bo‘lamiz.

5). *f*(-0.094)=-1,841<0, *f*(-0,125)=0,132>0 bo‘lgani uchun yechim [-0,125; -0.094] oraliqda yotadi va t_5=(-0,125-0,094)/2=-0,1095. bu yerda $|a_5 - b_5|$=0,031>2e=0,02, bo‘lgani uchun yechim [-0,125; -0,1095] oraliqda, *f*(-0,1095)=-0,00872<0, t_6=(-0,125-0,1095)/2=-0,11725, bundan *f*(-0,11725) = 0,0623, yechim [-0,1173;-0,1095] oraliqda bo‘ladi, bu yerda *f*(-0,11725)=0,0623, yechim [-0,1173, -0,1095] oraliqda bo‘ladi, bu yerda

|-0,1095 – (-0,1173)| = | 0,1173 – 0,1095| = 0,008<2e=0,02

bo‘lgani uchun taqribiy ildiz

$$x \approx \frac{-0{,}1095 - 0{,}1173}{2} = -0{,}1134 \approx -0{,}11$$

bo‘ladi.

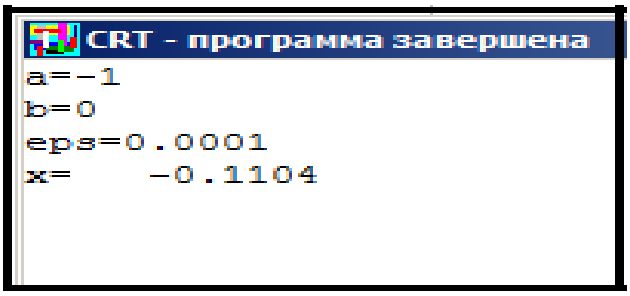
Quyida $e^x - 10x - 2 = 0$ tenglamani kesmani teng ikkiga bo’lish usuli bilan yechishning blok-sxemasi va ABC Pascal dasturlash tilida yozilgan dasturi keltirilgan:

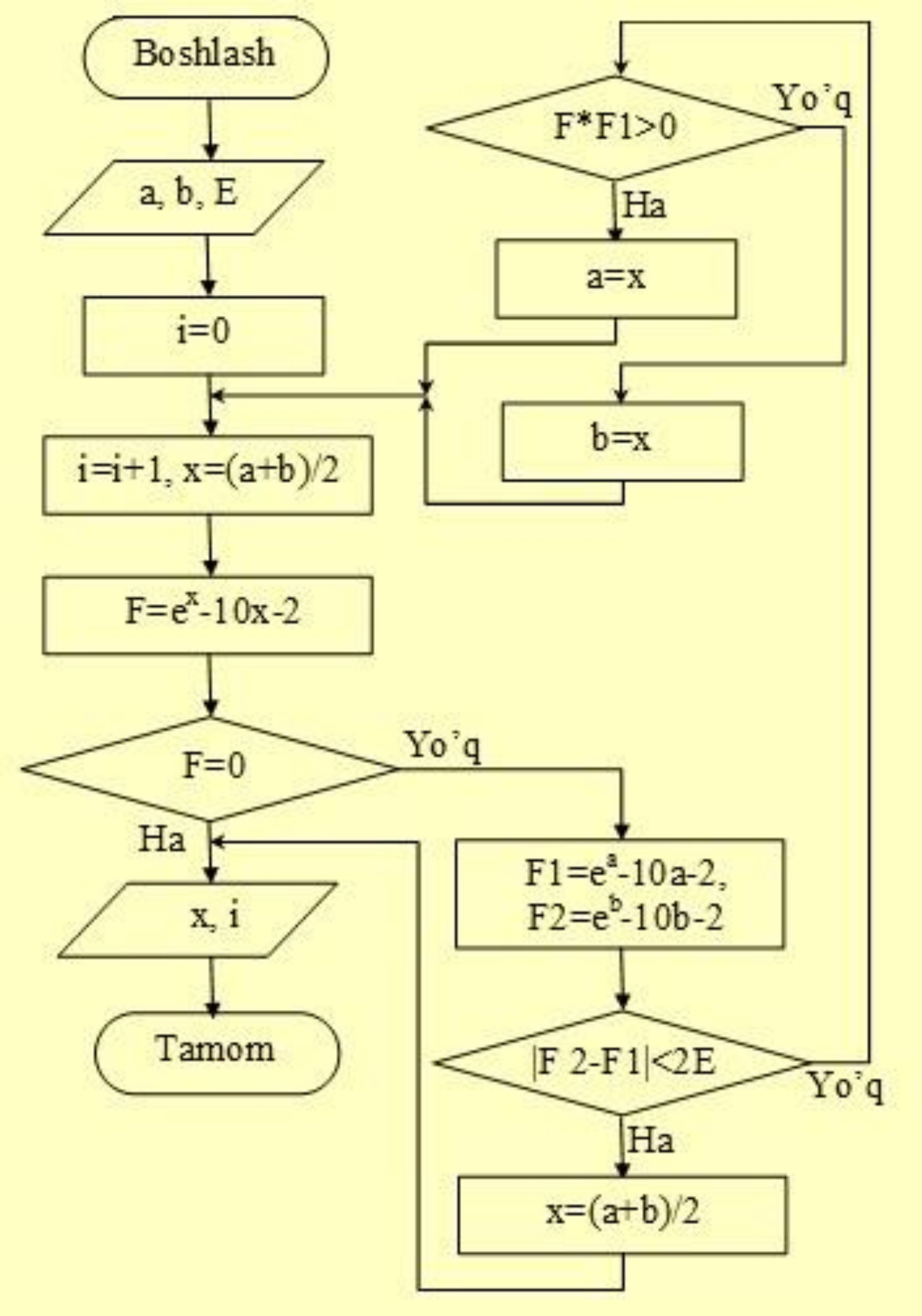
Dastur matni

```
program oraliq2; uses crt;
     var a,b,eps,x,fa,fc,c:real;
        function f(x:real):real;
        begin
     f:=exp(x)-10*x-2;
        end;
begin clrscr;
      write('a='); read(a);
      write('b='); read(b);
      write('eps='); read(eps);
      fa:=f(a);
            while abs(b-a)>eps do
            begin
            c:=(a+b)/2;
            fc:=f(c);
if fa*fc<=0 then b:=c else begin a:=c; fa:=fc end;
end;
     writeln('x=',c:10:4);
     end.
```

1.2-rasm. $e^x - 10x - 2 = 0$ tenglamasini kesmani teng ikkiga bo’lish usuli bilan yechish dasturi oynasining ko’rinishi.

Natija:





1.1-rasm. $e^x - 10x - 2 = 0$ **tenglamani kesmani teng ikkiga bo'lish usuli bilan yechishning blok-sxemasi.**

MUSTAQIL ISHLAR UCHUN TOPSHIRIQLAR

Quyidagi tenglamalar uchun:

1. Ildizlarning qisqa atrofini EHM yordamida aniqlang;
2. Aniqlangan oraliqda kesmani teng ikkiga bo‘lish usuli bilan E=0.000001 aniqlikda taqribiy hisoblang.

1). $\ln(x^2+1)-x^3-1=0$	15). $\ln(x^2+0{,}9)-2x^3-1=0$
2). $\ln(x^2+2)=x^4-3$	16). $\ln(x^2+2{,}2)=x^4-3{,}4$
3). $\ln(x^4+1)=8-x^6$	17). $\ln(x^4+1{,}5)=6{,}8-x^6$
4). $\ln(x^2+10)=10-x^4$	18). $\ln(x^2+8{,}8)=6{,}5-x^4$
5). $\ln(x^2+1)-10x^3+1=0$	19). $\ln(x^2+1{,}2)-9{,}2x^3+1{,}1=0$
6). $\ln(x^2+12)=x^4-5$	20). $\ln(1{,}2x^2+9)=1{,}3x^4-6$
7). $\ln(x^4+7)=7-x^6$	21). $\ln(x^4+4{,}7)=4{,}7-x^6$
8). $\ln(x^2+3)=4-4x+x^2$	22). $\ln(x^2+3{,}5)=1{,}3-9x+x^2$
9). $\ln(x^4+1)=2-x^2+5x$	23). $\ln(1{,}1x^4+1)=2{,}8-x^2+7x$
10). $\ln(x^2+5)=1+x^4+3x$	24). $\ln(x^2+4{,}5)=1{,}5+x^4+3{,}9x$
11). $\ln(x^2+1)-7x^3+2=0$	25). $\ln(1{,}9x^2+1)-10x^3-1=0$
12). $\ln(x^2+11)=x^4-3$	26). $\ln(1{,}3x^2+8{,}7)=2x^4-5$
13). $\ln(x^4+1)=17-x^6$	27). $\ln(x^4+6{,}1)=6{,}1-x^6$
14). $\ln(x^2+1)=4-9x+x^2$	28). $\ln(0{,}8x^2+2{,}8)=4{,}5-5x+x^2$

Chekli [*a*,*b*] oraliqda aniqlangan va uzluksiz *f(x)* funkiya berilgan bo‘lib, uning birinchi va ikkinchi tartibli hosilalari shu oraliqda mavjud bo‘lsin. Shu bilan birga [*a*,*b*] da *f’(x)* funksiya o‘z ishorasini saqlasin.

$$f(x)=0 \qquad (1)$$

tenglama [*a*,*b*] oraliqda yagona yechimga ega bo‘lsin va bu yechimni berilgan $\varepsilon>0$ aniqlikda topish talab qilingan bo‘lsin. Quyida bu yechimni aniqlash uchun bir necha sonli usullar, ularning Paskal algoritmik tilida tuzilgan programmalarni keltiramiz.

O‘z-o‘zini tekshirish uchun savollar

1. Yechim yotgan Kesmani aniqlash.
2. Boshlang‘ich shartni tanlash usulini tushuntiring.
3. Iteratsiya usulining yaqinlashish shartini ayting.
4. Iteratsiya usulida boshlang‘ich shartni tanlash usulini tushuntiring.
5. Kesmani ikkiga bo‘lish usuli va uning yaqinlashish shartini ayting.
6. Tenglamalarni taqribiy hisoblashda ketma-ket yaqinlashish (iteratsiya) shartlari.
7. Iteratsiya usulini qo‘llashda x=j(x) tenglamadagi j(x) uchun qo‘yilgan shartlar.

2-Laboratoriya ishi

Algebraik va transendent tenglamalarni yechishning oddiy vatarlar va urinmalar (Nyuton) usullari va ularning algoritmi

Ishdan maqsad: Algebraik va transendent tenglamalarni yechimini oddiy vatarlar va urinmalar (Nyuton) usullari va ularning algoritmi ishlab chiqish.

Vatarlar usuli. Aniqlik uchun $f(a)>0$ ($f(a)<0$) bo‘lsin. $A=A(a;f(a))$, $B=B(b;f(b))$ nuqtalardan to‘g’ri chiziq o‘tkazamiz va bu to‘g’ri chiziqni Ox o‘qi bilan kesishish nuqtasini $x_1 = a - \frac{b-a}{f(b)-f(a)} \cdot f(a)$ deb belgilaymiz. Agar $|a-x_1|\le\varepsilon$ bo‘lsa, $x=x_1$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi. Bu shart bajarilmasa, $b=x_1$ ($a=x_1$) deb olamiz. A, B nuqtalardan to‘g’ri chiziq o‘tkazamiz va uning Ox o‘qi bilan kesishish nuqtasini $x_2 = a - \frac{b-a}{f(b)-f(a)} \cdot f(a)$ deb olamiz. Agar $|x_2-x_1|\le\varepsilon$ shart bajarilsa, $x=x_2$ (1) tenglamaning ε aniqlikdagi taqribiy yechimi bo‘ladi, aks holda $b=x_2$ ($a=x_2$) deb olib, yuqoridagi amallar ketma-ketligini $|x_i-x_{i-1}|\le\varepsilon$ (i=3,4,…) shart bajarilguncha davom ettiramiz. Natijada (1) tenglamaning $x=x_i$ taqribiy yechimini hosil qilamiz.

x_n larning ketma-ket hisoblash formulasi quyidagi ko‘rinishga ega bo‘ladi:

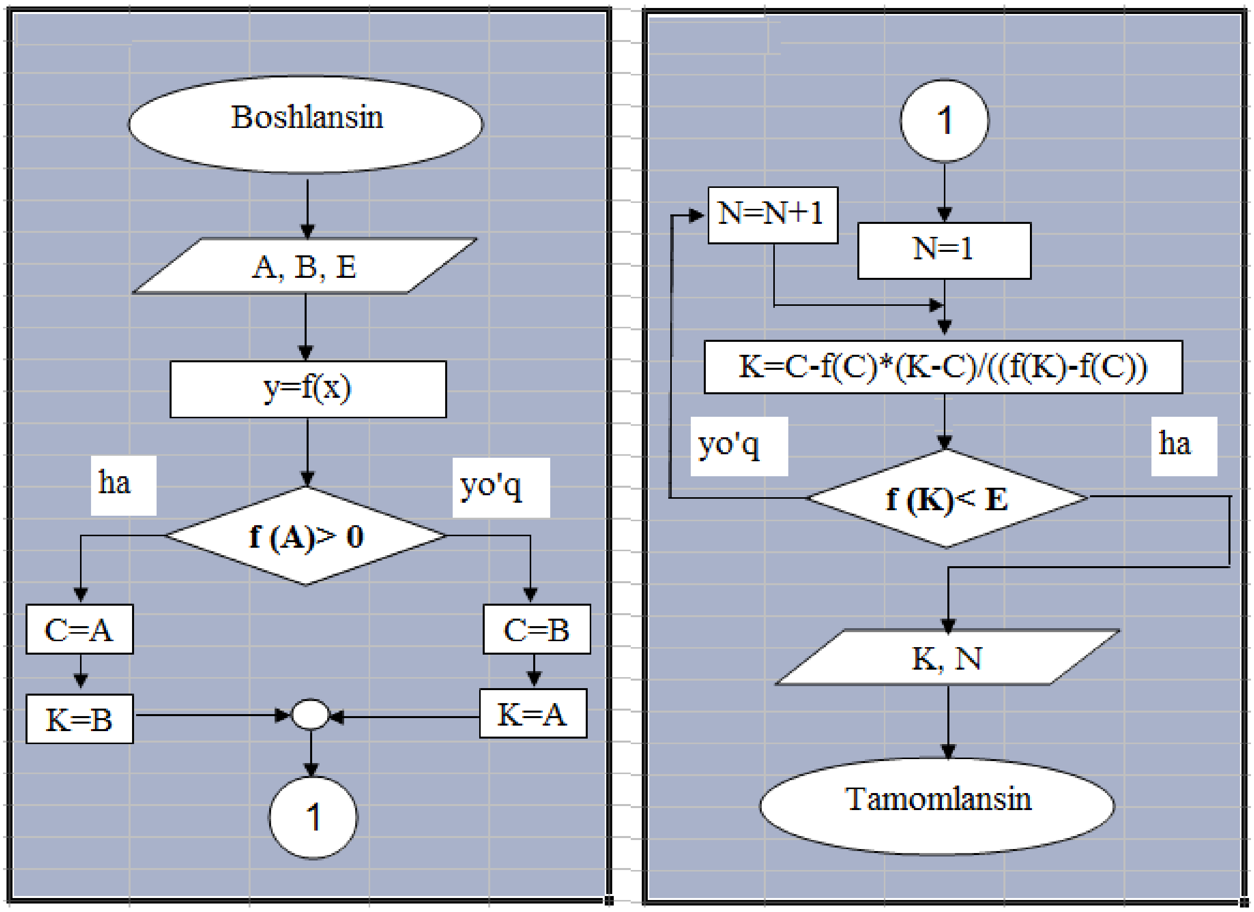
$$x_n = a - \frac{x_{n-1}-a}{f(x_{n-1})-f(a)} \cdot f(a) \quad \left(x_n = b - \frac{x_{n-1}-b}{f(x_{n-1})-f(b)} \cdot f(b) \right)$$

Misol. $tg(0{,}55x+0{,}1)-x^2=0$ tenglamaning [0,6;0,8] oraliqdagi ildizini ε=0,005 aniqlikda hisoblang.

Yechish. $|x_2-x_1|=0{,}002<\varepsilon$ bajariladi. x_2=0,7517; x_1=0,7417 bundan x=0,7517.

Berilgan tenglamaning taqribiy ildizini vatarlar usulida ε=0,0000000001 aniqlik bilan ABC Pascal dasturida hisoblaymiz.

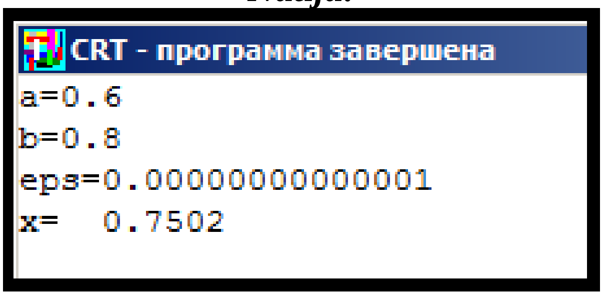
Yechish. Vatarlar usuli algoritmi blok-sxemasini keltiramiz:



Vatarlar usuliga Paskal tilida tuzilgan dasturning ko‘rinishi:

```
program vatar; uses crt;    {Vatarlar usuli}
   label 1,2;
   var a,b,eps,x:real;
    function f(x:real):real;
      begin
          f:= sin(0.55*x+0.1)/cos(0.55*x+0.1)-sqr(x);
      end;
   begin clrscr;
            write('a='); read(a);
              write('b='); read(b);
                    write('eps='); read(eps);
         2: x:=b;
             x:=b-f(b)*(b-a)/(f(b)-f(a));
             if abs(f(x))<eps then goto 1 else begin b:=x; goto 2 end;
         1: writeln('x=',x:8:4);
  end.
```

Natija:



Mustaqil yechish uchun amaliy topshiriqlar (variantlar)

Tenglamalarning vatarlar usuli yordamida 0.00000001 aniqlikda ildizini hisoblang

tenglama	kesma	tenglama	kesma
$x(5+\cos(7x-1))-4=0$	[0,8;1]	$\cos^2\left(1+\frac{1}{1+x^2}\right)-0,95x=0$	[0;0,2]
$e^{1-x}-x^5+1=0$	[1;1,2]	$x^2(3+\sin x^3)-5=0$	[1;1,2]
$x^{19}-x^2+25=0$	[-1,2;-1]	$\ln\left(1+\frac{8}{1+x^4}\right)-x^2+16=0$	[4;4,2]
$arctg(1+8x)+0,58x^5-1=0$	[0;0,2]	$x^9-x^2+16=0$	[-1,4;-1,2]
$x^{11}-\frac{1}{x^2}-1=0$	[1;1,2]	$\log_8(1+x^2)+x-1=0$	[0,6;0,8]
$\sin^2\left(1-\frac{1}{1+x^2}\right)-0,12x+0,2=0$	[7,4;7,6]	$x^7-x^2+4=0$	[-1,2;-1]
$9^{1+x}+5x=1$	[-0,6;-0,4]	$\log_5(1+x^4)+x-2=0$	[1,2;1,4]
$x^{21}-x^4+25=0$	[-1,2;-1]	$2\sin x^2+1=\frac{1}{10x^2+2}$	[1,8;2]
$2^x(5-\sin x)+x^3=0$	[-1,4;-1,2]	$x^5-x^2+9=0$	[-1,6;-1,4]
$3\cos x^2+0,5=\frac{1}{10x^2+1}$	[1,2;1,4]	$arctgx^2+0,5=\frac{1}{10x^2+1}$	[0,2;0,4]
$arctgx^2-1=\frac{1}{1+x^2}$	[1,6;1,8]	$\ln(x^2+7)-1=\frac{1}{1+x^2}$	[0,2;0,4]
$\ln(x^2+17)-7=\frac{1}{1+x^2}$	[32,8;33]	$\cos(1+x^2)-0,4=\frac{1}{x^2+5}$	[4;4,2]
$\cos(x^3-1)-0,3=\frac{1}{x^4+11}$	[-5,2;-5]	$(x^5-1)\sin(x+1)-3x=1$	[5,2;5,4]

<u>Nyuton usuli (Urinmalar usuli).</u> $[a,b]$ oraliqda $f'(x)$ va $f''(x)$ ning ishoralari o‘zgarmasdan qolsin. $f(x)$ funksiya grafigining $V=V(b,f(b))$ nuqtasidan urinma o‘tkazamiz. Bu urinmaning Ox o‘qi bilan kesishgan nuqtasini b_1 deb belgilaymiz. $f(x)$ funksiya grafigining $V_1=V_1(b_1,f(b_1))$ nuqtasidan yana urinma o‘tkazamiz va bu urinmaning Ox o‘qi bilan kesishgan nuqtasini b_2 deb belgilaymiz. Bu jarayonni bir necha marta takrorlab, $b_1,b_2,...,b_n$ larni hosil qilamiz. $|b_n-b_{n-1}|<\varepsilon$ shart bajarilganda hisoblash to‘xtatiladi. $b_i=b_{i-1}-\frac{f(b_{i-1})}{f'(b_{i-1})}$

Bu usulni ikki holat uchun ko’rib chiqamiz.

1- h o l a t . Faraz qilaylik, $f(a)<0, f(b)>0, f'(x)>0, f''(x)>0$ yoki $f(a)>0, f(b)<0, f'(x)<0, f''(x)<0$

Urinmaning tenglamasi quyidagicha:

$$y - f(b) = f'(b)(x-b),$$

bu yerda $y=0$, $x=x_1$ deb , (2.1) ni x_1 nisbatan yechsak,

$$x_1 = b - \frac{f(b)}{f'(b)}$$

Shu mulohazani [$a; x_1$] kesma uchun takrorlab, x_2 ni topamiz:

$$x_2 = x_1 - \frac{f(x)}{f'(x)}$$

Umuman olganda $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Hisoblashni $|x_{n+1} - x_n| \le \varepsilon$ shart bajarilganda to'xtatamiz.

2- holat. Faraz qilaylik f(a) < 0, f(b) > 0, f'(x) > 0, f''(x) < 0 yoki f(a)>0, f(b) < 0, f'(x) < 0, f''(x) > 0 . y = f(x) egri chiziqka A nuqtada urinma o'tkazamiz, uning tenglamasi: y - f(a) = f' (a) (x – a), bu yerda y=0, x=x1 decak,

$$x_1 = a - \frac{f(a)}{f'(a)}$$

[x1;b] kesmadan

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Umuman

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1-Misol. *x-sinx=0,25* tenglamaning ildizi ε=0,0001 aniqlikda urinmalar usuli bilan aniqlansin.

Y e c h i s h . Tenglamaning ildizi [0,982; 1,178] kesmada ajratilgan, bu yerda *a=0,982*; *b=1,178*;

[0,982; 1,178] kesmada *f'(x)=1-cosx x>0;* *f''(x) = sin x>0* , bo'lgani uchun 1- holat bo'yicha yechiladi. ($x_0 = b$)

[0,982; 1,178] kesmada boshlangich yaqinlashishda x_0 =1,178 olinadi. Keyingi hisoblashlarni tegishli formula vositasida bajaramiz.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{bunda } n = 0 \text{ bulsa,}$$

ya`ni: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1{,}1778 - \frac{1{,}1778 - \sin(1{,}1778) - 0{,}25}{1 - \cos(1{,}1778)} = 1{,}1715$

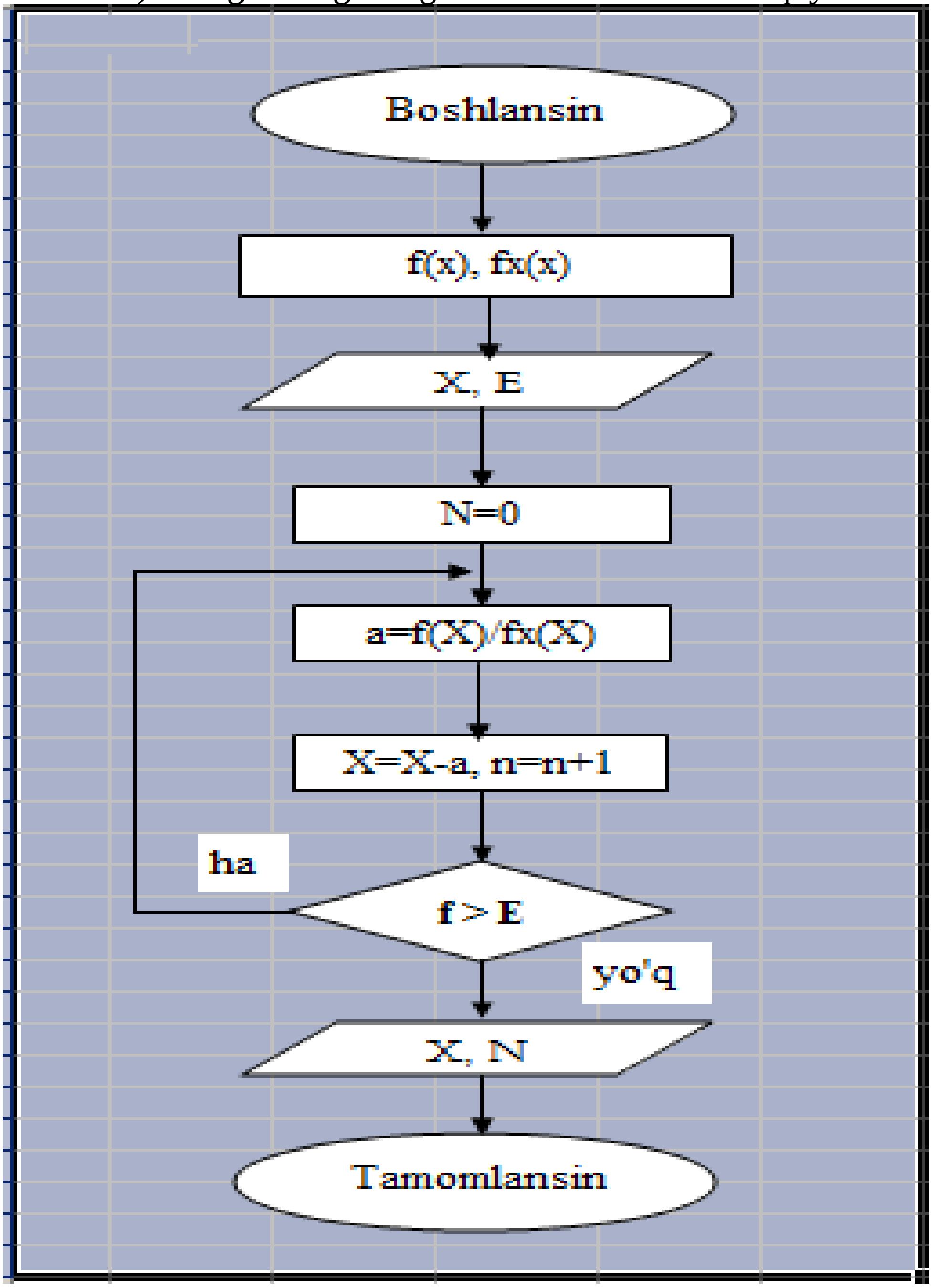
Hisoblash natijalarini quyidagi 1-jadvalda keltiramiz.

1 - jadval

n	x_n	$- \sin x_n$	$f(x_n)=x_n-\sin x_n-0{,}25$	$f'(x_n)=1-\mathrm{sos}x_n$	$\frac{f(x_n)}{f'(x_n)}$
0	1,178	- 0,92384	0,00416	0,61723	- 0,0065
1	1,1715	- 0,92133	0,00017	0,61123	- 0,0002
2	1,1713	- 0,92127	0,00003	0,61110	- 0,0005
3	1,17125				

Jadvaldan ko'rinadiki, $x_3 - x_2 = |1{,}17125 - 1{,}1713| = 0{,}00005 < \varepsilon$. Demak yechim deb $x = 1{,}17125$ ni ($\varepsilon = 0{,}0001$ aniqlikda) olish mumkin.

Nyuton (urinmalar) usuliga tuzilgan algoritm blok – sxemasini quyida keltirilgan:



2-Misol. $tg(0{,}55x+0{,}1)-x^2=0$ tenglamaning [0,6;0,8] oraliqdagi ildizini $\varepsilon=0{,}005$ aniqlikda hisoblang.

Yechish. $|x_2-x_1|=0{,}002\leq\varepsilon$; $x=x_2=0{,}7503$.

Urinmalar usuliga Paskal tilida tuzilgan dasturning ko'rinishi:

```
program urinma; uses crt;          {Urinmalar usuli}
    var x0,eps,x1,a:real; function f(x:real):real;
                begin
                      f:=        { f(x) funksiyasining ko'rinishi }
```

```
                end;
          function fx(x:real):real;
                begin
                      fx:=       { f'(x) funksiyasining ko'rinishi}
                end;
begin      clrscr;  write('x0='); read(x0);  write('eps='); read(eps);
           x1:=x0;  repeat
           a:=f(x1)/fx(x1);
          x1:=x1-a;
    until abs(f(x1))<eps;
          writeln('x=',x1:10:4);
end.
```

1-misolda berilgan tenglamaning 0,0001 aniqlikdagi ildizini ABC Pascal dasurida tuzilgan dasturda hisoblaymiz.

```
program urinma; uses crt;
        var x0,eps,x1,a:real;
function f(x:real):real;
              begin
 f:= x-sin(x)-0.25;
                end;
function fx(x:real):real;
              begin
      fx:=1-cos(x);
              end;
begin      clrscr;
               write('x0='); read(x0);
               write('eps='); read(eps);
               x1:=x0;
      repeat
               a:=f(x1)/fx(x1);
              x1:=x1-a;
      until abs(f(x1))<eps;
              writeln('x=',x1:5:5);
end.
```

Dastur natijasi:

```
CRT - программа завершена
x0=1.178
eps=0.0001
x=1.17126
```

Mustaqil bajarish uchun amaliy ish variantlari.

Berilgan tenglamalarni Nyuton usuli (Urinmalar usuli) da 0,000001 aniqlikda yeching.

Tenglama	**kesma**	**Tenglama**	**kesma**
$x\sin(5x+2)-x^2=-0,27$	[0,3;0,4]	$x\cos(10x-1)-x^4+1=0$	[-1;-0,9]
$e^x-e^{-x}-2=0$	[0;1]	$\frac{\ln 1,4}{3+x^2}-\sin x=0,16$	[3,2;3,3]
$3\sin\sqrt{x}+0,35x-3,8=0$	[2;3]	$x\sin(8x+1)-x^2=0,08$	[0;0,1]
$x-2+\sin\frac{1}{x}=0$	[1,2;2]	$1-x+\sin x-\ln(1+x)=0$	[0;1,5]
$x\sin(5x+8)-x^2=0,2$	[-0.5;-0.4]	$x-\frac{1}{3+\sin 3,6x}=0$	[0;0,85]
$x^2-\ln(1+x)-3=0$	[2;3]	$x\sin(11x+1)-x^4+1=0$	[0,8;0,9]
$\ln x-x+1,8=0$	[2;3]	$0,1x^2-x\ln(x+1)=0$	[1;2]
$x+\cos\left(x^{0,52}+2\right)=0$	[0,5;1]	$\sqrt{1-0,4x}-\arcsin x=0$	[0;1]
$x^2+10x-10=0$	[0;1]	$3x-4\ln x-5=0$	[2;4]
$0,4+arctg\sqrt{x}-x=0$	[1;2]	$\arccos x-\sqrt{1-0,3x^2=0}$	[0;1]
$2x-3\ln x-3=0$	[0,5;0,6]	$4+\arcsin\sqrt{x}-x=0$	[-1;5]
$0,4+tgx^2+x=0$	[-5;6]	$2-ctg\sqrt{x}-x^2=0$	[-4;5]
$0,4+\arcsin\sqrt{x}-x=0$	[-3;4]	$0,24-arcctg\sqrt{x}-x=0$	[-2;5]

Berilgan f(x)=0 tenglamani unga teng kuchli bo‘lgan x=φ(x) ko‘rinishdagi tenglamaga keltiramiz.

Teorema 2.1. Aytaylik,

1).. φ(x) funktsiya [a,b] oraliqda aniqlangan va differentsiallanuvchi bo‘lsin;

2).. φ(x) funktsiyaning hamma qiymatlari [a,b] oraliqqa tushsin;

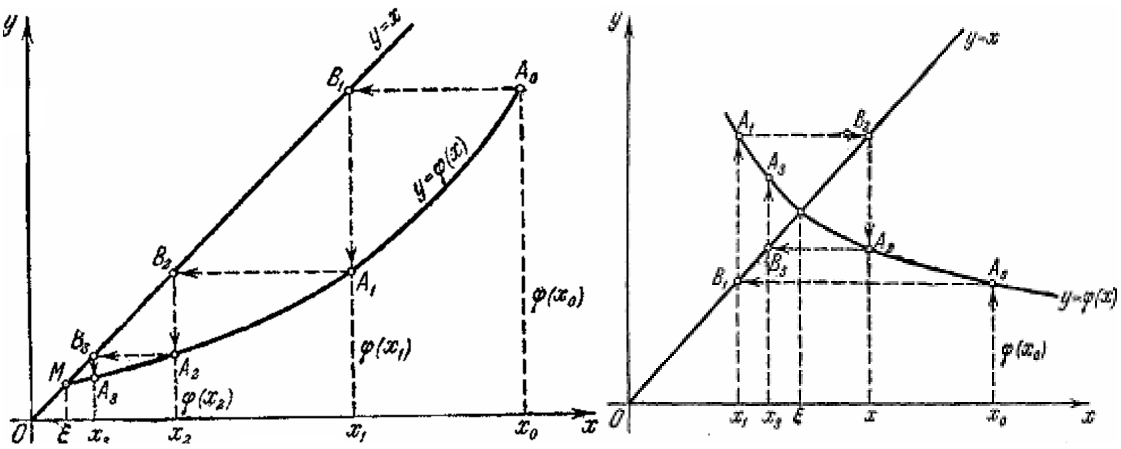
3)..[a,b] oraliqda|φ(x)|≤q<1 tengsizlik bajarilsin.

Bu holda [a,b] oraliqda x=φ(x) tenglamaning yagona x=t yechimi mavjud va bu yechim $t_0\in[a;b]$ qanday tanlanishidan qat’iy nazar

$$t_1=\varphi(t_0),\ t_2=\varphi(t_1),\ldots,\ t_n=\varphi(t_{n-1}),\ldots$$

formulalar bilan aniqlanadigan { t_n } ketma – ketlikning limitidan iborat bo‘ladi.

Berilgan f(x)=0 tenglamani unga teng kuchli bo‘lgan x=φ(x) tenglama uchun yaqinlashish sharti bajarilganda yaqinlashish jarayonini quyidagi shakillar misolida ko‘rish mumkin.



1-Rasm.

Bu yerda, t_0 qiymat [a,b] oraliqda yotuvchi ixtiyoriy son bo‘lib, yechimning 0-yaqinlashishi, t_i – n_i yechimning i – yaqinlashishi deb yuritiladi.

Bu teorema asosida tenglama ildizini quyidagicha aniqlaymiz.

1) $f(x)=0$ tenglamaning yagona ildizi yotgan [a,b] kesmani biror (masalan, grafik) usul bilan aniqlaymiz.

2) [a,b] da $f(x)$ ning uzluksizligi va $f(a)\cdot f(b)<0$ shart bajarilishini tekshiramiz.

3)Tenglamani $x=\varphi(x)$ ko‘rinishga keltirib, $\varphi(x)\in[a,b]$ ekanligini hamda [$a;b$] da $\varphi'(x)$ mavjudligini tekshiramiz va $q=\max\limits_{x\in[a;b]}|\varphi'(x)|$ ni topamiz.

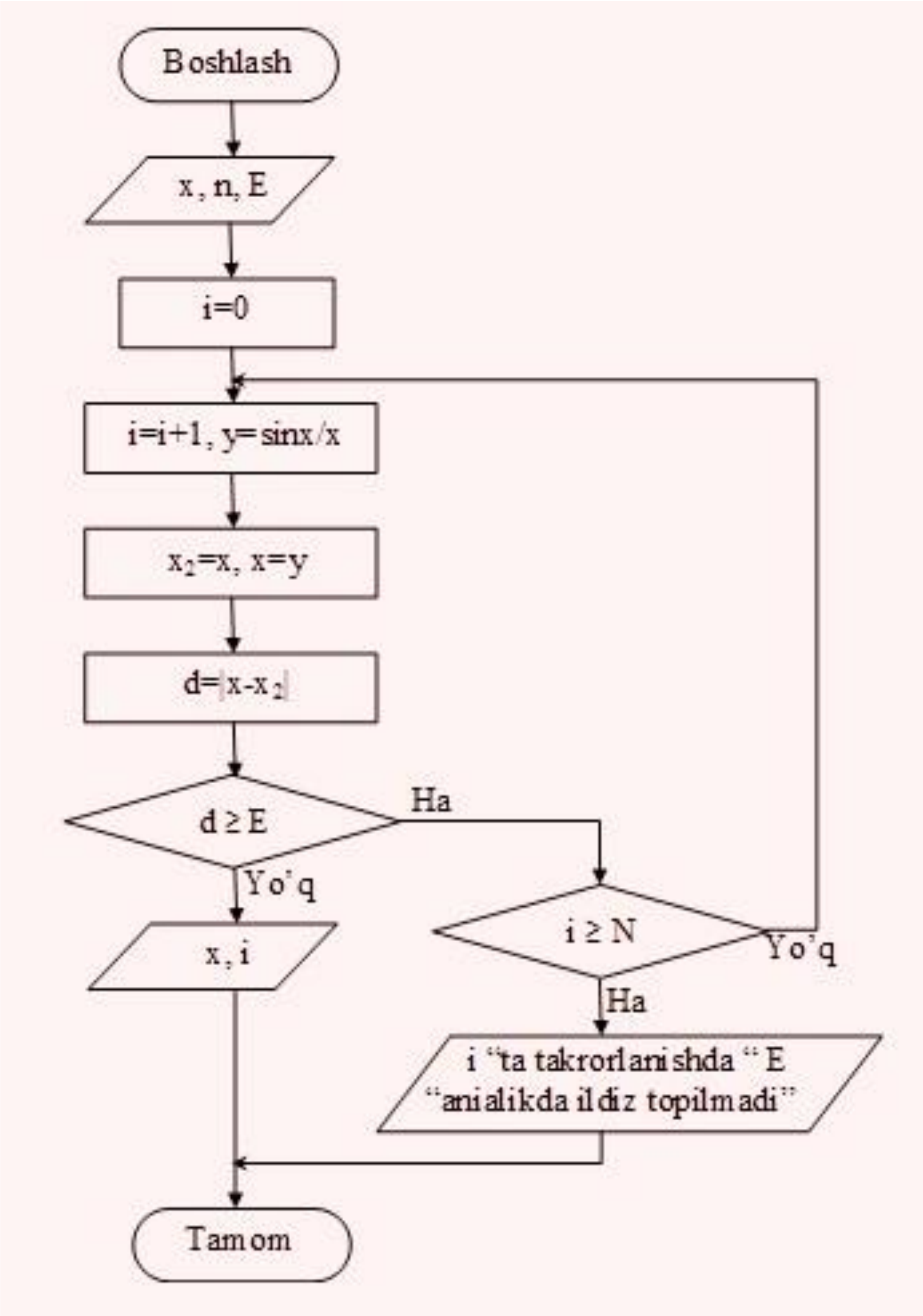
4) Agar q<1 bo‘lsa, $x_n=\varphi(x_{n-1})$ ketma-ketlikning boshlang‘ich yaqinlashishi x_0 uchun [$a;b$] ning ixtiyoriy bitta nuqtasi olamiz.

5) Ketma-ketlik hadlarini hisoblashni $|x_n- x_{n-1}|<\varepsilon(1-q)/q$ shart bajarilguncha davom ettiramiz.

6) Ildizning taqribiy qiymati uchun x_n ni olamiz.

Dastu matni:

```
Program   iter; uses crt;
Label  2;
Const eps = 0.00001;
VAR  x, y, del : real; n :integer;
begin
write(' Boshlangich qiymatni kiritish X0=');readln (x);
n:= 0;
2 : y:= sin(x)/x;
    del:= abs(y-x); x:=y;
    n:= n+1;
if del > eps then goto 2;
writeln('Tenglamaning taqribiy ildizi');
writeln ('x=', x);
writeln('iteratsiyalar soni n=', n);
END.
```



sinx-x^2=0 tenglamani iteratsiya usuli bilan yechisning blok-sxemasi

Tenglama ildizini saqlovchi oraliqni topamiz. Natijada [0,8;0,9] oraliqga tegishli ihtiyoriy nuqtani boshlang'ich yechim sifatida kiritish mumkin.

```
Program   iter; uses crt;
Label  2;
Const eps = 0.00001;
VAR  x, y, del : real; n :integer;
begin
write('бошлангич кийматни киритинг X0=');readln (x);
n:= 0;
2 : y:= sin(x)/x;
     del:= abs(y-x); x:=y;
     n:= n+1;
if del > eps then goto 2;
writeln('тенгламанинг такрибий илдизи');
writeln ('x=', x);
writeln('такрорланишлар сони n=', n);
END.
```

CRT - программа завершена

```
бошлангич кийматни киритинг X0=0.8
тенгламанинг такрибий илдизи
x=0.87672408977448
такрорланишлар сони n=8
```

2-misol. Tenglamaning ildizini ajrating va uni 0,00001 aniqlikda hisoblang:

$$\frac{\ln(1+|\sin x|)}{e^x+\cos^2 x}-\frac{1}{2x^2}=arctg\frac{x}{3}$$

Yechish. Ildizni ajratamiz:

```
program ildizni_ajratish; uses crt;
label 1,2;
     var a,c,x,fa,fc,h:real;
             i,M,N:integer;
        function f(x:real):real;
        begin
     f:=ln(1+abs(sin(x)))/(exp(x)+sqr(cos(x)))-1/2*sqr(x)-arctan(x/3);
        end;
begin clrscr;
     2: write('a='); read(a);
      write('N='); read(N);
       write('M='); read(M);
      h:=1/N;

begin
for i:=1 to M do begin c:=a+h; fa:=f(a);fc:=f(c);
if fa*fc<0 then goto 1;
a:=c;end; end; goto 2;
 1: writeln('a=',a:5:3);
  writeln('b=', c:5:3);
    end.
```

```
CRT - программа завершена
a=0
N=10
M=100
a=0.100
b=0.200
```

Demak, [0,1;0,2] oraliqda tenglama ildizi mavjud va uni hisoblaymiz:

```
Program  iter; uses crt;
Label  2;
Const eps = 0.00001;
VAR  x, y, del, t : real; n :integer;
begin
write('бошлангич киймати киритинг X0=');readln (x);
n:= 0;
2 : t:=ln(1+abs(sin(x)))/(exp(x)+sqr(cos(x)))-1/2*sqr(x);
y:= 3*sin(t)/cos(t);
     del:= abs(y-x); x:=y;
     n:= n+1;
if del > eps then goto 2;
writeln('тенгламанинг такрибий илдизи');
writeln ('x=', x);
writeln('такрорланишлар сони n=', n);
END.
```

```
CRT - программа завершена
бошлангич киймати киритинг X0=0.1
тенгламанинг такрибий илдизи
x=0.179581940350565
такрорланишлар сони n=16
```

Mustaqil bajarish uchun topshiriqlar.

1. Tenglamalarning ildizini ajrating va uni 0,00001 aniqlikda iterasiya usulida yechish algoritm blok-sxemasini va ildizni hisoblash dasturini tuzing. Ildizni berilgan aniqlikda hisoblang.

1). $\frac{\ln(1+|0,2x-\cos^2 x|)}{e^x+\sin^2 x}-\frac{1}{1+2x^4}=arctg\frac{x}{K}$ *(bu yerda K=3,5,7,9,11,13,15,17)*

2). $12,01x^5+1,89x^4-0,98x^2+4521x=1,789$

3).

1.	1) $2^x+5x-3=0$ 2) $3x^4-4x^3-12x^2-5=0$ 3) $0.5^x+1=(x-2)^2$	**2.**	1) $arctgx-1/(3x^3)=0$ 2) $2x^3-9x^2-60x+1=0$ 3) $[\log_2(-x)](x+2)=-1$
3.	1) $5^x+3x=0$ 2) $x^4-x-1=0$ 3) $0.5^x+x^2=2$ 4) $(x-1)^2Ln(x+1)=1$	**4.**	1) $2e^x=2+5x$ 2) $2x^4-x^2-10=0$ 3) $xLog_3(x+1)=1$ 4) $Cos(x+0.5)=x^3$
5.	1) $3^{x-1}-2-x=0$ 2) $3x^4+8x^3+6x^2-10=0$ 3) $(x-4)^2\log_{0.5}(x-3)=-1$	**6.**	1) $arctgx-1/(2x^3)=0$ 2) $x^4-18x^2+6=0$ 3) $x^22^x=1$
7.	1) $e^{-2x}-2x+1=0$ 2) $x^4+4x^3-8x^2-17=0$ 3) $0.5^x-1=(x+2)^2$ 4) $x^2Cos2=-1$	**8.**	1) $5^x-6x-3=0$ 2) $x^4-x^3-2x^2+3x-3=0$ 3) $0.5^x-2x^2-3=0$ 4) $xLog(x+1)=1$
9.	1) $arctg(x-1)+2x=0$ 2) $3x^4+4x^3-12x^2+1=0$ 3) $(x-2)^22^x=1$ 4) $x^2-20Sinx=0$	**10.**	1) $2arctgx-x+3=0$ 2) $3x^4-8x^3-18x^2+3=0$ 3) $2Sin(x+1.1)=0.5x^2-1$ 4) $2Logx-x/2+1=0$
11	1) $3^x+2x-2=0$ 2) $2x^4-8x^3+8x^2-1=0$ 3) $[(x-2)^2-1]2^x=1$ 4) $(x-2)Cosx=1$	**12.**	1) $2arctgx-3x+2=0$ 2) $2x^4+8x^3+8x^2-1=0$ 3) $Sin(x-0.5)-x+0.8=0$ 4) $(x-1)Log_2(x+2)=1$
13.	1) $3^x+2x-5=0$ 2) $x^4-4x^3-8x^2+1=0$ 3) $0.5^x+x^2-3=0$ 4) $(x-2)^2Lg(x+1)=1$	**14.**	1) $2e^x+3x+3x+1=0$ 2) $3x^4+4x^3-12x^2-5=0$ 3) $Cos(x+0.3)=x^2$ 4) $xLog_3(x+1)=2$
15.	1) $3^{x-1}-4-x=0$ 2) $2x^3-9x^2-60x+1=0$ 3) $(x-3)^2Log_{0.5}(x-2)=-1$ 4) $Sinx=x-1$	**16.**	1) $arctgx-1/(3x^3)=0$ 2) $x^4-x-1=0$ 3) $(x-1)^22^x=1$ 4) $tg^3x=x-1$
17.	1) $e^x+x+1=0$ 2) $2x^4-x^2-1=0$ 3) $0.5^x-3=(x+2)^2$ 4) $(x-2)^22^x=1$	**18.**	1) $3^x-2x+5=0$ 2) $3x^4+8x^3+6x^2-10=0$ 3) $2x^2-0.5^x=0$ 4 $xLg(x+1)=1$
19	1) $arctg(x-1)+3x-2=0$	**20.**	1) $2arctgx-x+3=0$

.	2) x^4-18x^2+6=0 3) x^2-20Sinx=0		2) x^4+4x^3-8x^2 -17=0 4) 2Lgx-x/2+1=0
21	1) 2^x-3x-2=0. 2)x^4-x^3-2x^2 +3x- 3=0; 3)$(0.5)^x$+ 1=$(x-2)^2$ 4)(x-3)cosx = -1, -2’<x<2’.	**22**	1) arctgx+2x-1=0 2) 3x^4+4x^3-12x^2 +1=0 3)(x+2)Log_2(x)=1 4) Sin(x+1)=0.5x
23	1) 3^x+2x-3=0. 2) 3x^4-8x^3-18x^2 +2=0; 3) $(0.5)^x$=4-x^2 4) $(x+2)^2$Lg(x+11)=1	**24**	1) 2e^x-2x-3=0. 2) 3x^4+4x^3-12x^2 -5=0; 3) xLog_2(x+1)=1 4) cos(x+0.5)= x^3
25	1) 3^x+2+x=0. 2) 2x^3-9x^2 -60x+1=0; 3) $(x-4)^2$$Log_{0.5}$(x-3)=-1 4) 5Sinx=x-0.5	**26**	1) arctg(x-1)+2x-3=0 2) x^4x -1=0; 3) $(x-1)^2$$2^x$=1 4) x^2-10sinx = 0
27	1) 2e^x-2x-3=0. 2) 2x^4-x^2 -10=0; 3) $(0.5)^x$-3=-$(x+1)^2$ 4) x^2cos2x = 1	**28**	1) 3^x-2x-5=0. 2) 3x^4+8x^3+6x^2 -10=0; 3) 2x^2 -0.5^x - 3=0 4) xLg(x+1)=1

Nazorat savollari

1. Iteratsiya usulining mohiyatini aytib bering.
2. Tenglama iteratsiya usulini qo’llash uchun qanday ko’rinishga olib keltiriladi.
3. Iteratsiya usulining yaqinlashish shartining ma’nosini aytib bering.
4. Iteratsiya usulining nazariy va amaliy xatoliklarining ma’nosini aytib bering.
5. Vatarlar usulida yaqinlashish shartini tushuntiring?
6. Vatarlar usulining yaqinlashish formulasini keltirib chiqaring?
7. Algoritmni blok-sxemada tasvirlash afzalligi va kamchiligi nimada?

3-Laboratoriya ishi

Chiziqli algebraik tenglamalar sistemasini yechishning Gauss usuli va uning algoritmi

Ishdan maqsad: Talabalarga chiziqli algebraik va transendent tenglamalar sistemasini Gauss, usulida yechish algoritmlarini berish hamda bu usullarga Paskal tilida tuzilgan dasturda ishlashga o‘rgatish.

Nazariy qism

Bizga n ta noma’lumli n ta chiziqli algebraik tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n} = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n} = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \qquad (1)$$

berilgan bo‘lsin. Bu yerda a_{ij}, b_i lar berilgan sonlar, x_i lar noma’lumlar (i,j=1,2,...,n). Agar (1) sistemaga mos keluvchi asosiy determenant 0 dan farqli, ya’ni

$$\begin{vmatrix} a_{11}\ a_{12} \ldots a_{1n} \\ a_{21}\ a_{22} \ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots \\ a_{n1}\ a_{n2} \ldots a_{nn} \end{vmatrix} \neq 0$$

bo‘lsa u yagona yechimga ega bo‘ladi.

Chiziqli algebraik tenglamalar sistemasini yechishning bir necha usullari mavjud bo‘lib, ulardan asosiylari Kramer, Gauss, teskari matritsa, iteratsiya usullaridir. Bu usullardan Gauss usuli bilan yechish algoritmini (1) sistema uchun ko‘rib chiqaylik.

__**Gauss usuli.**__ Gauss usuli yoki no’malumlarni ketma-ket yo‘qotish usuli chiziqli algebraik tenglamalar sistemasini aniq yechish usuli hisoblanadi. Bu usulining algoritmi quyidagi hisoblashlar ketma-ketligidan iborat.

$a_{11} \neq 0$ bo‘lsin (agar $a_{11} = 0$ bo‘lsa, sistemadagi tenglamalarning o‘rnini almashtirib $a_{11} \neq 0$ ga ega bo‘lish mumkin). (1) sistemadagi birinchi tenglamaning barcha hadlarini a_{11} ga bo‘lib

$$x_1 + a_{12}^{(1)}x_2 + \ldots + a_{1n}^{(1)}x_n = b_1^{(1)}$$

ni hosil qilamiz. Bu tenglamani ketma-ket $a_{21}, a_{31}, \ldots, a_{n1}$ larga ko‘paytirib, undan sistemaning keyingi tenglamalarini ayiramiz va

$$
\begin{cases}
x_1 + a_{12}^{(1)}x_2 + \ldots + a_{1n}^{(1)}x_n = b_1^{(1)} \\
a_{22}^{(1)}x_2 + \ldots + a_{2n}^{(1)} = b_2^{(1)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
a_{n2}^{(1)}x_2 + \ldots + a_{nn}^{(1)}x_n = b_n^{(1)}
\end{cases}
\quad (2)
$$

sistemaga ega bo‘lamiz. Bu yerda $a_{ij}^{(1)} = a_{ij} - \frac{a_{i1}a_{1j}^{(1)}}{a_{11}}$, $b_j^{(1)} = b_j - \frac{b_1 a_{j1}}{a_{11}}$ i=2,…,n; j=2,3,…,n.

(2) sistema uchun yuqoridagi hisoblashlar (noma’lumlarni ketma-ket yuqotish) ni bir necha bor takrorlab, quyidagi

$$
\begin{cases}
x_1 + a_{12}^{(1)}x_2 + a_{13}^{(1)}x_3 + \ldots + a_{1n}^{(1)}x_n = b_1^{(1)} \\
x_2 + a_{23}^{(2)}x_3 + \ldots + a_{2n}^{(2)}x_n = b_2^{(2)} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
x_n = b_n^{(n)}
\end{cases}
\quad (3)
$$

sistemani hosil qilamiz va x_i larni topish uchun

$$
x_k = a_{k,n-1}^{(k-1)} - \sum_{j:=k+1}^{n} a_{kj}^{(k-1)}x_j , k = n, n-1, n-2, \ldots, 1
$$

formulaga ega bo‘lamiz.

Chiziqli algebraik tenglamalar sistemasini Gauss usulida yechish uchun Paskal algoritmik tilida tuzilgan dastur matni.

```
Program gauss; uses crt;
const n=4;      {tenglamalar soni}
type
stroka=array[1..n] of real;
matrisa=array[1..n, 1..n+1] of real;
vektor=array[1..n] of real;
var
a:matrisa; x:vektor; max,c:real;
i,j,k,m:integer;
procedure gauss_1(b:matrisa; var y:vektor);
begin
              for i:=1 to n do
                begin          max:=abs(b[i,i]); j:=i;
       for k:=i+1 to n do if abs(b[k,i])>max then
                begin max:=abs(b[k,i]);
                   j:=k; end;
                  if j<>i then for k:=i to n+1 do
       begin c:=b[i,k]; b[i,k]:=b[j,k];
                   b[j,k]:=c; end;
```

```
c:=b[i,i];          for k:=i to n+1 do b[i,k]:=b[i,k]/c;
     for m:=i+1 to n do
begin   c:=b[m,i];
     for k:=i+1 to n+1 do    b[m,k]:=b[m,k]-b[i,k]*c;
          end;   end;
     y[n]:=b[n,n+1];
  for i:=n-1 downto 1 do
begin
y[i]:=b[i,n+1];
for k:=i+1 to n do y[i]:=y[i]-b[i,k]*y[k]
end;    end;
      begin  clrscr;
for i:=1 to n do
for j:=1 to n+1 do
begin
 write('a[',i:1,',',j:1,']=');
read(a[i,j]); end;
gauss_1(a,x);
writeln( 'Sistemaning yechimi' );
for i:=1 to n do writeln('x[',i:1,']=',x[i]:10:4);       end.
```

Misol. Berilgan chiziqli algebraik tenglamalar sistemalarini Gauss usuli yordamida yeching.

$$\begin{cases} 5x_1 - x_2 + x_3 - 7x_4 = 10 \\ 2x_1 - 8x_2 - 31x_4 = 22 \\ 3x_1 - 11x_3 + x_4 = -33 \\ 2x_1 + 2x_2 - 2x_3 - 50x_4 = -60 \end{cases}$$

Yechish. Berilgan dastur matnidan foydalanib hisoblaymiz :

```
program gauss; uses crt;
const n=4;       {tenglamalar soni}
type   stroka=array[1..n] of real;
matrisa=array[1..n, 1..n+1] of real; vektor=array[1..n] of real;
var a:matrisa; x:vektor; max,c:real; i,j,k,m:integer;
procedure gauss_1(b:matrisa; var y:vektor);
begin for i:=1 to n do begin max:=abs(b[i,i]); j:=i;
for k:=i+1 to n do if abs(b[k,i])>max then  begin max:=abs(b[k,i]);
j:=k; end; if j<>i then for k:=i to n+1 do begin c:=b[i,k]; b[i,k]:=b[j,k];
b[j,k]:=c; end; c:=b[i,i];for k:=i to n+1 do b[i,k]:=b[i,k]/c;
for m:=i+1 to n do begin c:=b[m,i];   for k:=i+1 to n+1 do
b[m,k]:=b[m,k]-b[i,k]*c;  end;   end;  y[n]:=b[n,n+1];  for i:=n-1 downto 1 do
 begin  y[i]:=b[i,n+1]; for k:=i+1 to n do y[i]:=y[i]-b[i,k]*y[k]
end; end;  begin  clrscr; for i:=1 to n do for j:=1 to n+1 do begin
 write('a[',i:1,',',j:1,']=');  read(a[i,j]); end;  gauss_1(a,x);
writeln( 'Sistemaning yechimi' );  for i:=1 to n do
writeln('x[',i:1,']=',x[i]:10:4); end.
```

Dastur natijasi

```
CRT - программа завершена
a[1,2]=-1
a[1,3]=1
a[1,4]=-7
a[1,5]=10
a[2,1]=0
a[2,2]=2
a[2,3]=-8
a[2,4]=-31
a[2,5]=22
a[3,1]=3
a[3,2]=0
a[3,3]=-11
a[3,4]=1
a[3,5]=-33
a[4,1]=2
a[4,2]=2
a[4,3]=-2
a[4,4]=-50
a[4,5]=-60
Sistemaning yechimi
x[1]= -502.2581
x[2]=-2008.9355
x[3]= -142.4839
x[4]=  -93.5484
```

Topshiriq

1-masala. Berilgan chiziqli algebraik tenglamalar sistemalarini Gauss usuli yordamida yeching.

1. $\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 1 \\ 2x_1 - x_2 - 3x_4 = 2 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 - 5x_4 = -6 \end{cases}$

2. $\begin{cases} x_1 + 5x_2 = 2 \\ 2x_1 - x_2 + 3x_3 + 2x_4 = 4 \\ 3x_1 - x_2 - x_3 + 2x_4 = 6 \\ 3x_1 - x_2 + 3x_3 - x_4 = 6 \end{cases}$

3. $\begin{cases} x_1 + x_2 - x_3 - x_4 = 0 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = -1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$

4. $\begin{cases} x_1 - 4x_2 = 2 \\ x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 2x_1 + 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$

5. $\begin{cases} 5x_1 + x_2 - x_4 = 9 \\ 3x_1 - 3x_2 - x_3 + 4x_4 = -1 \\ 3x_1 - 2x_3 + x_4 = -16 \\ x_1 - 4x_2 + x_4 = 0 \end{cases}$

6. $\begin{cases} 5x_1 - x_2 + x_3 + 3x_4 = -4 \\ x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ 2x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + x_2 + x_3 + 2x_4 = 4 \end{cases}$

7. $\begin{cases} 2x_1 + x_3 + 4x_4 = 9 \\ x_1 + 2x_2 - x_3 + x_4 = 8 \\ 2x_1 + x_2 + x_3 + x_4 = 5 \\ x_1 - x_2 + 2x_3 + x_4 = -1 \end{cases}$

8. $\begin{cases} 4x_1 - 2x_2 + x_3 - 4x_4 = 3 \\ 2x_1 - x_2 + x_3 - x_4 = 1 \\ 3x_1 - x_3 + x_4 = -3 \\ 2x_1 + 2x_2 - 2x_3 + 5x_4 = -6 \end{cases}$

9. $\begin{cases} 2x_1 - 6x_2 + 2x_3 + 2x_4 = 12 \\ x_1 + 3x_2 + 5x_3 + 7x_4 = 12 \\ 3x_1 + 5x_2 + 7x_3 + x_4 = 0 \\ 5x_1 + 7x_2 + x_3 + 3x_4 = 4 \end{cases}$

10. $\begin{cases} 2x_1 - x_3 - 2x_4 = -1 \\ x_2 + 2x_3 - x_4 = 2 \\ x_1 - x_2 - x_4 = 1 \\ -x_1 + 3x_2 - 2x_3 = 0 \end{cases}$

11. $\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 - x_2 - x_3 - 2x_4 = -4 \\ 2x_1 + 3x_2 - x_3 - x_4 = -6 \\ x_1 + 2x_2 + 3x_3 - x_4 = -4 \end{cases}$

12. $\begin{cases} x_1 + 5x_2 + 3x_3 - 4x_4 = 20 \\ 3x_1 + x_2 - 2x_3 = 9 \\ 5x_1 + 7x_2 + 10x_4 = -9 \\ 3x_2 - 5x_3 = 1 \end{cases}$

13. $\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 6 \\ x_1 - x_2 - 2x_3 - 3x_4 = 8 \\ 3x_1 + 2x_2 - x_3 + 2x_4 = 4 \\ 2x_1 - 3x_2 + 2x_3 + x_4 = -8 \end{cases}$

14. $\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8 \\ x_1 - 3x_2 - 6x_4 = 9 \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0 \end{cases}$

15. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$

16. $\begin{cases} 3x_1 + 2x_2 - 2x_3 + 4x_4 = 7 \\ 2x_1 - x_2 + 2x_3 - 3x_4 = -5 \\ -x_1 + 2x_2 - x_3 + 12x_4 = 1 \\ -7x_1 + x_2 + 2x_3 + x_4 = 3 \end{cases}$

Nazorat savollari

1. Algebraik va transendent tenglamalar sistemalarini yechishning qanday usullarini bilasiz.
2. Algebraik va transendent tenglamalar sistemalarini yechishning Gauss usuli nima?
3. Algebraik va transendent tenglamalar sistemalarini yechishning Gauss usuliga algoritm tuzing?
4. Algebraik va transendent tenglamalar sistemalarini yechishda qaysi holatda Gauss usulini qo'llash mumkin?

4-Laboratoriya ishi

Chiziqli algebraik tenglamalar sistemasini yechishning oddiy iteratsiya, Zeydel usullari va ularning algoritmi

Ishdan maqsad : Talabalarga chiziqli algebraik va transendent tenglamalar sistemasini oddiy iteratsiya va Zeydel usullarida yechish algoritmlarini berish hamda bu usullarga Paskal tilida tuzilgan dasturda ishlashga o‘rgatish.

Nazariy qism

Bizga n ta noma'lumli n ta chiziqli algebraik tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n} = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n} = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases} \quad (1)$$

berilgan bo‘lsin. Bu yerda a_{ij}, b_i lar berilgan sonlar, x_i lar noma'lumlar (i,j=1,2,...,n). Agar (1) sistemaga mos keluvchi asosiy determenant 0 dan farqli, ya'ni

$$\begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix} \neq 0$$

bo‘lsa u yagona yechimga ega bo‘ladi.

Noma’lumlar soni ko’p bo’lganda chiziqli tenglamalar sistemasini yechishning Kramer, Gauss, teskari matrisa usullari bilan olinishi ancha murakkab bo’lib qoladi. Bunday hollarda taqribiy sonli usullardan foydalanish ancha samarali hisoblanadi. Shunday usullardan biri oddiy iterasiya usulidir.

Quyidagi tenglamalar sistemasi berilgan bo’lsin.

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \quad i = 1,2,...,n \qquad (6.1)$$

Bu sistema matrisa ko’rinishida quyidagicha yoziladi:

$$Ax = b,$$

Bu yerda

$$A = \begin{pmatrix} a_{11} & a_{12} & .. & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}, \ B = \begin{pmatrix} b_1 \\ b_2 \\ ... \\ b_n \end{pmatrix}, \ \ x = \begin{pmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{pmatrix}.$$

(6.1) da $a_{ii} \neq 0$ $(i=1,n)$ deb faraz qilamiz.

Tenglamalar sistemasida 1-tenglamani x_1 ga nisbatan, 2- tenglamani x_2 ga nisbatan, va ohirgisini x_n ga nisbatan yechamiz:

$$\begin{cases} x_1 = \beta_1 + 0 + \alpha_{12}x_2 + \alpha_{13}x_3 + ... + \alpha_{1n}x_n \\ x_2 = \beta_2 + \alpha_{21}x_2 + 0 + \alpha_{23}x_3 + ... + \alpha_{2n}x_n \\ ...... \\ x_n = \beta_n + \alpha_{n1}x_2 + \alpha_{n2}x_2 + \alpha_{n3}x_3 + ... + \alpha_{nn-1}x_{n-1} + 0 \end{cases} \qquad (6.2)$$

Ushbu

$$\alpha = \begin{pmatrix} 0 & \alpha_{12} & ... & \alpha_{in} \\ \alpha_{21} & 0 & ... & \alpha_{2n} \\ ... & ... & ... & ... \\ \alpha_{n1} & a_{n2} & ... & 0 \end{pmatrix} \quad \text{va} \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ ... \\ \beta_n \end{pmatrix}$$

Matrisalar yordamida (6.2) ni quyidagicha yozish mumkin: $x = \beta + \alpha x$ (6.3)

(6.3) sistemani ketma-ket yaqinlashish usuli bilan yechamiz.

$$x^{(0)}=\beta, \quad x^{(1)} = \beta + \alpha x^{(0)}, \ x^{(2)} = \beta + \alpha x^{(1)}, ...$$

Bu jarayonni quyidagicha ifodalaymiz:

$$x^{(k)} = \beta + \alpha x^{(k-1)}, \ x^{(0)}=\beta \qquad (6.4)$$

Bu ketma-ketlikning limiti, agar u mavjud bo’lsa (6.1) sistemaning yechimi bo’ladi.

Biz

$$x^{(k)} = \begin{pmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \dots \\ x_n^{(k)} \end{pmatrix}$$

belgilashni kiritamiz. Agar ihtiyoriy $\varepsilon>0$ uchun $\left|x_i^{(k+1)} - x_i^{(k)}\right| < \varepsilon$ tengsizlik barcha $i=1,2,...n$ uchun bajarilsa $x^{(k+1)} = \left(x_1^{(k+1)}, x_2^{(k+1)}, ..., x_n^{(k+1)}\right)$ мулещк (6,1) sistemaning ε aniqlikdagi yechimi deb yuritiladi.

Teorema. Agar keltirilgan (6.2) system uchun $\sum_{j=1}^{n}\left|\alpha_{ij}\right| < 1$ yoki $\sum_{i=1}^{n}\left|\alpha_{ij}\right| < 1$ shartlardan birortasi bajarilsa, uholda (6.4) iterasiyon jarayon boshlang'ich yaqinlashishni tanlashga bog'liq bo'lmagan holda yagona yechimga yaqinlashadi.

Natija (6.4) tenglamalar sistemasi uchun

$$\sum_{\substack{j\neq 1 \\ j=1}}^{n} |a_{ij}| < |a_{11}|, \ \sum_{\substack{j\neq 2 \\ j=1}}^{n} |a_{2j}| < |a_{22}|, \ ..., \ \sum_{\substack{j\neq n \\ j=1}}^{n} |a_{nj}| < |a_{nn}|$$

tengsizliklar bajarilsa (6.4) iterasiya yaqinlashuvchi bo'ladi.

Misol. Tenglamalar sistemasini ε=0,001 aniqlikda oddiy iterasiya usuli bilan yeching:

$$\begin{cases} 4x_1 + 0{,}24x_2 - 0{,}08x_3 = 8 \\ 0{,}09x_1 + 3x_2 - 0{,}15x_3 = 9 \\ 0{,}04x_1 - 0{,}08x_2 + 4x_3 = 20 \end{cases}$$

Yechish:

$$\left.\begin{array}{l} 0{,}24 + |-0{,}08| = 0{,}32 < |a_{11}| = 4 \\ 0{,}09 + |-0{,}15| = 0{,}24 < |a_{22}| = 3 \\ 0{,}04 + |0{,}08| = 0{,}12 < |a_{33}| = 4 \end{array}\right\}$$

Demak, iterasiya yaqinlashuvchi.

$$\begin{cases} x_1 = 2 - 0{,}06x_2 + 0{,}02x_3 \\ x_2 = 3 - 0{,}03x_1 + 0{,}05x_3 \\ x_3 = 5 - 0{,}01x_1 + 0{,}02x_2 \end{cases}.$$

Nolinchi yaqinlashish: $x^{(0)} = \beta = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, $x_1^{(0)} = 2, x_2^{(0)} = 3, x_3^{(0)} = 5.$

$$\alpha = \begin{pmatrix} 0 & -0{,}06 & 0{,}02 \\ -0{,}03 & 0 & 0{,}05 \\ -0{,}01 & 0{,}02 & 0 \end{pmatrix}$$

(6.4) formula yordamida hisoblashlarni bajaramiz.

$$x^{(1)}=\beta+\alpha x^{(0)}=\begin{pmatrix}1{,}92\\3{,}19\\5{,}04\end{pmatrix};\quad x_1^{(1)}=1{,}92;\ x_2^{(1)}=3{,}19;\ x_3^{(0)}=5{,}04.$$

$$x^{(2)}=\beta+\alpha x^{(1)}=\begin{pmatrix}1{,}9094\\3{,}1944\\5{,}0446\end{pmatrix};\quad x^{(3)}=\beta+\alpha x^{(2)}=\begin{pmatrix}1{,}90923\\3{,}19495\\5{,}04485\end{pmatrix};$$

$$x_1^{(2)}=1{,}9094;\ x_2^{(2)}=3{,}1944;\ x_3^{(2)}=5{,}0446.$$

Ushbu jadval hosil bo'ladi.

Yaqinlashishlar (*k*)	x_1	x_2	x_3	$x_1^{(k)}-x_1^{(k-1)}$	$x_2^{(k)}-x_2^{(k-1)}$	$x_3^{(k)}-x_3^{(k-1)}$
0	2	3	5	-	-	-
1	1,92	3,19	5,04	0,08	0,19	0,04
2	1,9094	3,1944	5,0446	0,0106	0,0044	0,0046
3	1,90923	3,19495	5,04485	0,00017	0,00055	0,00025

Bunda $\left|x_1^{(3)}-x_1^{(2)}\right|=0{,}00017<\varepsilon$, $\left|x_2^{(3)}-x_2^{(2)}\right|=0{,}00055<\varepsilon$, $\left|x_3^{(3)}-x_3^{(2)}\right|=0{,}00025<\varepsilon$ bajariladi. $x=x^{(3)}$ ChTS ning taqribiy ildizi.

Tenglamalar sistemasini oddiy iterasiya usulida yechish uchun ABC Pascal algortmik tilida tuzilgan dastur matni.

```
program iter_sis; uses crt;
  label 1,2;   const n=3; {tenglamalar coni}
      type          matrisa=array[1..n,1..n] of real;
         vektor=array[1..n] of real;
      var    a,a1:matrisa; x,x0,b,b1:vektor; eps,s:real; i,j,k:integer;
  begin       clrscr;
      for i:=1 to n do begin
      for j:=1 to n do begin     write('a[',i:1,',',j:1,']='); read(a[i,j]) end;
                        write('b[',i:1,']='); read(b[i]);   end;
      eps:=0.0001;      for i:=1 to n do begin
                        b1[i]:=b[i]/a[i,i];
                        for j:=1 to n do a1[i,j]:=-a[i,j]/a[i,i] end;
                        for i:=1 to n do begin
                                   x0[i]:=b1[i];    a1[i,i]:=0;       end;
         2: for i:=1 to n do   Begin       s:=0.0;
                            for j:=1 to n do s:=s+a1[i,j]*x0[j];
                            x[i]:=b1[i]+s;   end;           k:=0;
           for i:=1 to n do if abs(x[i]-x0[i])<eps
               then begin k:=k+1; if k=n then goto 1 end
               else begin for j:=1 to n do x0[j]:=x[j]; goto 2 end;
    1:  writeln('Sistemaning taqribiy yechimi:');
      for i:=1 to n do writeln('x[',i:1,']=',x[i]:10:8);
```

end.

```
program iter_sis; uses crt;
  label 1,2;
  const n=3;    type
            matrisa=array[1..n,1..n] of real;
            vektor=array[1..n] of real;
  var  a,a1:matrisa; x,x0,b,b1:vektor; eps,s:real; i,j,k:integer;
  begin    clrscr;
       for i:=1 to n do begin
       for j:=1 to n do begin
  write('a[',i:1,',',j:1,']='); read(a[i,j]) end;
  write('b[',i:1,']='); read(b[i]);  end;   eps:=0.0001;
for i:=1 to n do begin      b1[i]:=b[i]/a[i,i];
for j:=1 to n do a1[i,j]:=-a[i,j]/a[i,i];  end;
for i:=1 to n do begin  x0[i]:=b1[i];  a1[i,i]:=0;  end;
  2: for i:=1 to n do   begin
  s:=0.0;
  for j:=1 to n do s:=s+a1[i,j]*x0[j];
  x[i]:=b1[i]+s;   end;
         k:=0;
         for i:=1 to n do if abs(x[i]-x0[i])<eps
         then begin k:=k+1; if k=n then goto 1 end
         else begin for j:=1 to n do x0[j]:=x[j]; goto 2 end;
     1:  writeln('Sistemaning taqribiy yechimi:');
       for i:=1 to n do writeln('x[',i:1,']=',x[i]:10:8);
 end.
```

Dastur natijasi

```
CRT - программа завершена
a[1,1]=4
a[1,2]=0.24
a[1,3]=-0.08
b[1]=8
a[2,1]=0.09
a[2,2]=3
a[2,3]=-0.15
b[2]=9
a[3,1]=0.04
a[3,2]=-0.08
a[3,3]=4
b[3]=20
Sistemaning taqribiy yechimi:
x[1]=1.90919900
x[2]=3.19496286
x[3]=5.04480668
```

Chiziqli algebraik tenglamalar sistemasining ildizlarini 0,0000001 aniqlikda iterasiya usulida taqribiy hisoblang.

№1

$$\begin{cases} 5.34X_1 + 0.71X_2 + 0.63X_3 = 2.8 \\ 0.71X_1 - 6.65X_2 - 0.18X_3 = 0.17 \\ 1.17X_1 - 2.35X_2 + 8.75X_3 = 1.28 \end{cases}$$

№2

$$\begin{cases} 3{,}75X_1 - 0{,}28X_2 + 0{,}17X_3 = 0{,}75 \\ 2{,}11X_1 - 5{,}11X_2 - 0{,}12X_3 = 1{,}11 \\ 0{,}22X_1 - 3{,}17X_2 + 11{,}81X_3 = 0{,}05 \end{cases}$$

№3

$$\begin{cases} 8{,}021X_1 - 0{,}18X_2 + 0{,}75X_3 = 0{,}11 \\ 0{,}13X_1 + 7{,}75X_2 - 0{,}11X_3 = 2{,}00 \\ 3{,}01X_1 - 0{,}33X_2 + 10{,}11X_3 = 0{,}13 \end{cases}$$

№4

$$\begin{cases} 9{,}031X_1 - 0{,}28X_2 + 0{,}05X_3 = 0{,}11 \\ 0{,}183X_1 + 7{,}71X_2 - 0{,}19X_3 = 5{,}00 \\ 0{,}01X_1 - 0{,}33X_2 + 3{,}11X_3 = 0{,}19 \end{cases}$$

№5

$$\begin{cases} 3{,}01,\ X_1 - 0{,}14X_2 - 0{,}15X_3 = 1{,}00 \\ 1{,}11X_1 + 8{,}13X_2 - 0{,}75X_3 = 0{,}13 \\ 0{,}17X_1 - 2{,}11X_2 + 4{,}71X_3 = 0{,}17 \end{cases}$$

№6

$$\begin{cases} 2{,}92X_1 - 0{,}83X_2 + 0{,}62X_3 = 2{,}15 \\ 0{,}24X_1 - 2{,}54X_2 + 0{,}43X_3 = 0{,}62 \\ 0{,}73X_1 - 0{,}81X_2 - 4{,}67X_3 = 0{,}88 \end{cases}$$

№7

$$\begin{cases} 5{,}24X_1 - 0{,}87X_2 - 3{,}17X_3 = 0{,}46 \\ 2{,}11X_1 - 6{,}45X_2 + 1{,}44X_3 = 1{,}5 \\ 0{,}48X_1 + 1{,}25X_2 - 3{,}63X_3 = 0{,}35 \end{cases}$$

№8

$$\begin{cases} 7{,}64X_1 - 0{,}83X_2 + 4{,}2X_3 = 2{,}23 \\ 0{,}58X_1 - 2{,}83X_2 - 1{,}43X_3 = 1{,}710 \\ 0{,}86X_1 + 0{,}77X_2 + 2{,}88X_3 = -0{,}54 \end{cases}$$

№9

$$\begin{cases} 1{,}32X_1 - 0{,}42X_2 + 0{,}85X_3 = 1{,}32 \\ 0{,}63X_1 - 1{,}43X_2 - 0{,}58X_3 = -0{,}44 \\ 0{,}84X_1 - 2{,}23X_2 - 3{,}52X_3 = 0{,}62 \end{cases}$$

№10

$$\begin{cases} 2{,}73X_1 - 1{,}24X_2 - 0{,}38X_3 = 0{,}58 \\ 1{,}25X_1 + 6{,}66X_2 - 0{,}78X_3 = 0{,}66 \\ 0{,}75X_1 + 1{,}22X_2 - 7{,}83X_3 = 0{,}92 \end{cases}$$

№11

$$\begin{cases} 1{,}62X_1 - 0{,}44X_2 - 0{,}86X_3 = 0{,}68 \\ 0{,}83X_1 + 1{,}42X_2 - 0{,}56X_3 = 1{,}24 \\ 0{,}58X_1 - 0{,}37X_2 - 1{,}62X_3 = 0{,}87 \end{cases}$$

№12

$$\begin{cases} 1{,}26X_1 - 0{,}34X_2 + 0{,}17X_3 = 3{,}14 \\ 0{,}75X_1 + 1{,}84X_2 - 0{,}48X_3 = -1{,}17 \\ 0{,}44X_1 - 1{,}85X_2 + 11{,}16X_3 = 1{,}83 \end{cases}$$

№13

$$\begin{cases} 3{,}46X_1 + 1{,}72X_2 + 0{,}53X_3 = 2{,}44 \\ 1{,}53X_1 - 5{,}32X_2 - 1{,}83X_3 = 2{,}83 \\ 0{,}75X_1 + 0{,}86X_2 + 3{,}72X_3 = 1{,}06 \end{cases}$$

№14

$$\begin{cases} 2{,}47X_1 + 0{,}65X_2 + 0{,}88X_3 = 1{,}24 \\ 1{,}34X_1 + 8{,}17X_2 + 2{,}54X_3 = 2{,}35 \\ 0{,}86X_1 - 1{,}73X_2 - 3{,}08X_3 = 3{,}15 \end{cases}$$

№15

$$\begin{cases} 4{,}24X_1 + 1{,}73X_2 - 1{,}55X_3 = 1{,}87 \\ 0{,}34X_1 + 5{,}27X_2 + 3{,}15X_3 = 2{,}16 \\ 3{,}05X_1 - 1{,}05X_2 + 6{,}63X_3 = -1{,}25 \end{cases}$$

№16

$$\begin{cases} 2{,}43X_1 + 1{,}04X_2 - 0{,}58X_3 = 2{,}71 \\ 0{,}74X_1 + 1{,}83X_2 + 0{,}17X_3 = 1{,}26 \\ 1{,}43X_1 - 1{,}58X_2 + 3{,}83X_3 = 1{,}03 \end{cases}$$

№17

$$\begin{cases} 2{,}43X_1 + 0{,}63X_2 + 1{,}44X_3 = 2{,}18 \\ 1{,}64X_1 - 5{,}83X_2 - 2{,}45X_3 = 1{,}84 \\ 0{,}58X_1 + 0{,}58X_2 + 3{,}18X_3 = 0{,}74 \end{cases}$$

№18

$$\begin{cases} 1{,}94X_1 + 0{,}62X_2 - 0{,}95X_3 = 1{,}43 \\ 2{,}15X_1 - 4{,}18X_2 + 0{,}57X_3 = 2{,}43 \\ 1{,}72X_1 - 0{,}83X_2 + 3{,}57X_3 = 3{,}88 \end{cases}$$

№19

$$\begin{cases} 4{,}62X_1 + 0{,}56X_2 - 0{,}43X_3 = 1{,}16 \\ 1{,}32X_1 - 5{,}88X_2 + 1{,}76X_3 = 2{,}07 \\ 0{,}73X_1 + 1{,}42X_2 - 3{,}34X_3 = 2{,}38 \end{cases}$$

№20

$$\begin{cases} 1{,}06X_1 + 0{,}34X_2 + 0{,}26X_3 = 1{,}17 \\ 2{,}54X_1 - 14{,}16X_2 + 0{,}55X_3 = 2{,}53 \\ 1{,}34X_1 - 0{,}47X_2 - 3{,}83X_3 = 3{,}26 \end{cases}$$

№21

$$\begin{cases} 4{,}21X_1 + 1{,}13X_2 - 1{,}45X_3 = 2{,}87 \\ 0{,}34X_1 + 8{,}27X_2 + 3{,}15X_3 = 2{,}14 \\ 3{,}05X_1 - 1{,}05X_2 + 6{,}31X_3 = -0{,}25 \end{cases}$$

№22

$$\begin{cases} 3{,}43X_1 + 1{,}04X_2 - 0{,}58X_3 = 1{,}71 \\ 0{,}74X_1 + 2{,}83X_2 + 0{,}17X_3 = 2{,}26 \\ 1{,}43X_1 - 1{,}58X_2 + 9{,}83X_3 = 4{,}03 \end{cases}$$

№23

$$\begin{cases} 5{,}43X_1 + 0{,}63X_2 + 1{,}44X_3 = 2{,}08 \\ 1{,}64X_1 - 4{,}83X_2 - 2{,}45X_3 = 1{,}94 \\ 0{,}58X_1 + 0{,}58X_2 + 2{,}18X_3 = 1{,}74 \end{cases}$$

№24

$$\begin{cases} 2{,}94X_1 + 1{,}62X_2 - 0{,}15X_3 = 1{,}53 \\ 0{,}15X_1 - 2{,}18X_2 + 0{,}57X_3 = 3{,}43 \\ 1{,}72X_1 - 0{,}83X_2 + 7{,}57X_3 = 3{,}18 \end{cases}$$

№25

$$\begin{cases} 2{,}62X_1 + 0{,}56X_2 - 0{,}43X_3 = 1{,}16 \\ 1{,}32X_1 - 4{,}88X_2 + 1{,}76X_3 = 2{,}97 \\ 1{,}73X_1 + 1{,}42X_2 - 4{,}34X_3 = 5{,}38 \end{cases}$$

№26

$$\begin{cases} 1{,}96X_1 + 0{,}34X_2 + 0{,}26X_3 = 2{,}17 \\ 2{,}54X_1 - 5{,}16X_2 + 0{,}55X_3 = 2{,}03 \\ 1{,}34X_1 + 1{,}47X_2 - 3{,}11X_3 = 11{,}26 \end{cases}$$

Zeydel usuli chiziqli bir qadamli birinchi tartibli ityeratsion usuldir. Bu usul oddiy ityeratsion usuldan shu bilan farq qiladiki, dastlabki yaqinlashish $x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}$ga ko’ra $x_1^{(1)}$ topiladi. So’ngra $x_1^{(1)}, x_2^{(0)}, \ldots, x_n^{(0)}$ ko’ra $x_2^{(1)}$ topiladi va x.k. Barcha $x_1^{(1)}$ lar aniqlangandan so’ng $x_i^{(2)}, x_i^{(3)}, \ldots$lar topiladi. Aniqroq aytganda, hisoblashlar quyidagi sxema bo’yicha olib boriladi:

$$x_1^{(k+1)} = \frac{b_1}{a_{11}} - \sum_{j=2}^{n} \frac{a_{1j}}{a_{11}} x_j^{(k)} \qquad x_2^{(k+1)} = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x_1^{(k+1)} - \sum_{j=3}^{n} \frac{a_{2j}}{a_{22}} x_j^{(k)}$$

$$x_i^{(k+1)} = \frac{b_i}{a_{ii}} = \frac{b_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{(k+1)} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{(k)} \qquad x_n^{(k+1)} = \frac{b_n}{a_{nn}} - \sum_{j=1}^{n-1} x_j^{(k+1)}$$

Ko’pincha Zeydel usuli oddiy ityeratsiya usuliga nisbatan yaxshiroq yaqinlashadi, ammo har doim ham bunday bo’lavyermaydi. Bundan tashqari Zeydel usuli programmalashtirish uchun qulaydir, chunki $x_i^{(k+1)}$ ning qiymati hisoblanayotganda $x_1^{(k)}, \ldots, x_{i-1}^{(k)}$ larning qiymatini saqlab qolishning hojati yo’q.

Misol. Zeydel usuli bilan misolning yyechimi 5 xona aniqlikda topilsin.

$$\begin{cases} 10x_1 + x_2 - 3x_3 - 2x_4 + x_5 = 6 \\ -x_1 + 25x_2 - x_3 - 5x_4 - 2x_5 = 11 \\ 2x_1 + x_2 - 20x_3 + 2x_4 - 3x_5 = -19 \\ x_2 - x_3 + 10x_4 - 5x_5 = 10 \\ x_1 + 2x_2 - x_3 - 2x_4 - 20x_5 = -32 \end{cases}$$

Y e c h i s h . Bu tizimning tenglamalarini mos ravishda 10, 25, - 20, 10, 20 larga bo’lib, quyidagi ko’rinishda yozib olamiz:

$$\begin{cases} x_1 = 0,6 - 0,1x_2 + 0,3x_3 + 0,2x_4 - 0,1x_5 \\ x_2 = 0,44 + 0,04x_1 - 0,04x_3 + 0,2x_4 + 0,08x_5 \\ x_3 = 0,95 + 0,1x_1 + 0,05x_2 + 0,1x_4 - 0,15x_5 \\ x_4 = 1 - 0,1x_2 + 0,1x_3 + 0,5x_5 \\ x_5 = 1,6 + 0,05x_1 + 0,1x_2 + 0,05x_3 + 0,1x_4 \end{cases}$$

bu yerda $\sum_{\substack{j=1 \\ j\neq i}}^{n} \left|\frac{a_{ij}}{a_{ii}}\right| \leq a < 1 \quad (i = 1,2,...,n)$ shart bajariladi. Haqiqadan ham,

$$\sum_{j=1}^{5} |C_{1j}| = -0,1 + 0,3 + 0,2 - 0,1 = 0,3 < 1; \qquad \sum_{j=1}^{5} |C_{2j}| = 0,28 < 1;$$

$$\sum_{j=1}^{5} |C_{3j}| = 0,41 < 1; \qquad \sum_{j=1}^{5} |C_{4j}| = 0,5 < 1;$$

$$\sum_{j=1}^{5} |C_{5j}| = 0,3 < 1;$$

Dastlabki yaqinlashish $x^{(0)}$ sifatida ozod hadlar ustuni (0,6; 0,44; 0,95; 1; 1,6)
Ityeratsiyaning birinchi qadamini bajaramiz:

$$x_1^{(1)} = 0,6 - 0,1\, x_2^{(0)} + 0,3x_3^{(0)} + 0,2x_4^{(0)} - 0,1x_5^{(0)} =$$
$$= 0,6 - 0,1 \cdot 0,44 + 0,3 \cdot 0,95 + 0,2 \cdot 1 - 0,1 \cdot 1,6 = 0,881$$
$$x_2^{(1)} = 0,44 + 0,04\, x_1^{(1)} - 0,04x_3^{(0)} + 0,2x_4^{(0)} + 0,08x_5^{(0)} =$$
$$= 0,44 + 0,04 \cdot 0,881 - 0,04 \cdot 0,95 + 0,2 \cdot 1 - 0,08 \cdot 1,6 = 0,771$$
$$x_3^{(1)} = 0,95 + 0,1\, x_1^{(1)} + 0,05x_2^{(1)} + 0,1x_4^{(0)} - 0,1x_5^{(0)} =$$
$$= 0,95 + 0,1 \cdot 0,881 + 0,05 \cdot 0,771 + 0,1 \cdot 1 - 0,15 \cdot 1,6 = 0,937$$
$$x_4^{(1)} = 1 - 0,1\, x_2^{(1)} + 0,1x_3^{(1)} + 0,5x_5^{(0)} = 1,817$$
$$x_5^{(1)} = 1,6 + 0,05x_1^{(1)} + 0,1x_2^{(1)} + 0,05x_3^{(1)} + 0,1x_4^{(1)} = 1,948$$

Keyingi yaqinlashishlarni 6- jadvalda keltiramiz:

6 - jadval

k	$x_1^{(k)}$	$x_2^{(k)}$	$x_3^{(k)}$	$x_4^{(k)}$	$x_5^{(k)}$
0	0,6	0,44	0,95	1	1,6
1	0.881	0,771	0,937	1,817	1,948
2	0,973	0,961	0,985	1.974	1,992
3	0,995	0,995	0,999	1,996	1,999
4	0,9995	0,9991	0,9997	1,9995	1,9998
5	0,99992	0,99989	0,99997	1.99991	1,99997
6	0,99999	0,99998	0,99999	1,99999	2.00000

Javob: $x_1 = x_2 = x_3 = 1;$ $x_4 = x_5 = 2$

Taqribiy hisoblashlar kompyuter texnologiyasi yordamida oson bajariladi. Buning uchun amaliy dasturlarga yoki dasturlashtirish tillariga murojaat etiladi. Quyida Turbo Paskal dasturlash tilida iteratsiya usuliga tuzilgan dastur matni:

```
uses crt;
```

```
type mat=array[1..20,1..20] of real;    vector=array[1..30] of real;
var  ag,temp,a,y,b,z,a2: mat;  temp2:array[1..20,1..20] of integer;
 i,p,q,j,k,n,nn,t: integer;  aa,aas,d,m,x,r,bg,x3,x2: vector;  ii: integer;
 m2,s2,max,l,s,f: real;  h: integer;
 begin   write('Count N=');    readln(n);
  for i:=1 to n do     begin      for j:=1 to n do       begin
       write('a[',i,'][',j,']=');       readln(a[i,j]);      end;
     write('b[',i,']=');      readln(a[i,j+1]);     end;
For i:=1 To n do For j:=1 To n+1 do  a2[i,j]:=a[i,j]/a[i,i];
For i:=1 To n do x[i]:= a2[i,n+1]; repeat
For i:=1 To n do  begin s:=a2[i,n+1]; For j:=1 To n do begin
If j<i Then s:=s-a2[i,j]*x3[j]; If j>i Then s:=s-a2[i,j]*x[j];
end; x2[i]:=x3[i]; x3[i]:=s; end; f:=0; For i:=1 To n do
begin  If Abs(x3[i]-x2[i])>0.00001 Then f:=1; x[i]:=x3[i]; end;
until f<>1;  writeln('Ziydel ildizlari'); for k:=1 to n do
writeln('X[',k,']=', x[k]:5:5); for t:=1 to n do begin
l:=a[t,t]; for j:=1 to n+1 do a[t,j]:=a[t,j]/l; for i:=t+1 to n do begin l:=a[i,t];
for j:=1 to n+1 do a[i,j]:=a[i,j]-a[t,j]*l ; end;
end;
end.
```

Yuqorida berilgan tenglamalar sitemasini Ziydel usulida yechishni ABCPascal dasturi yordamida amalga oshiramiz.

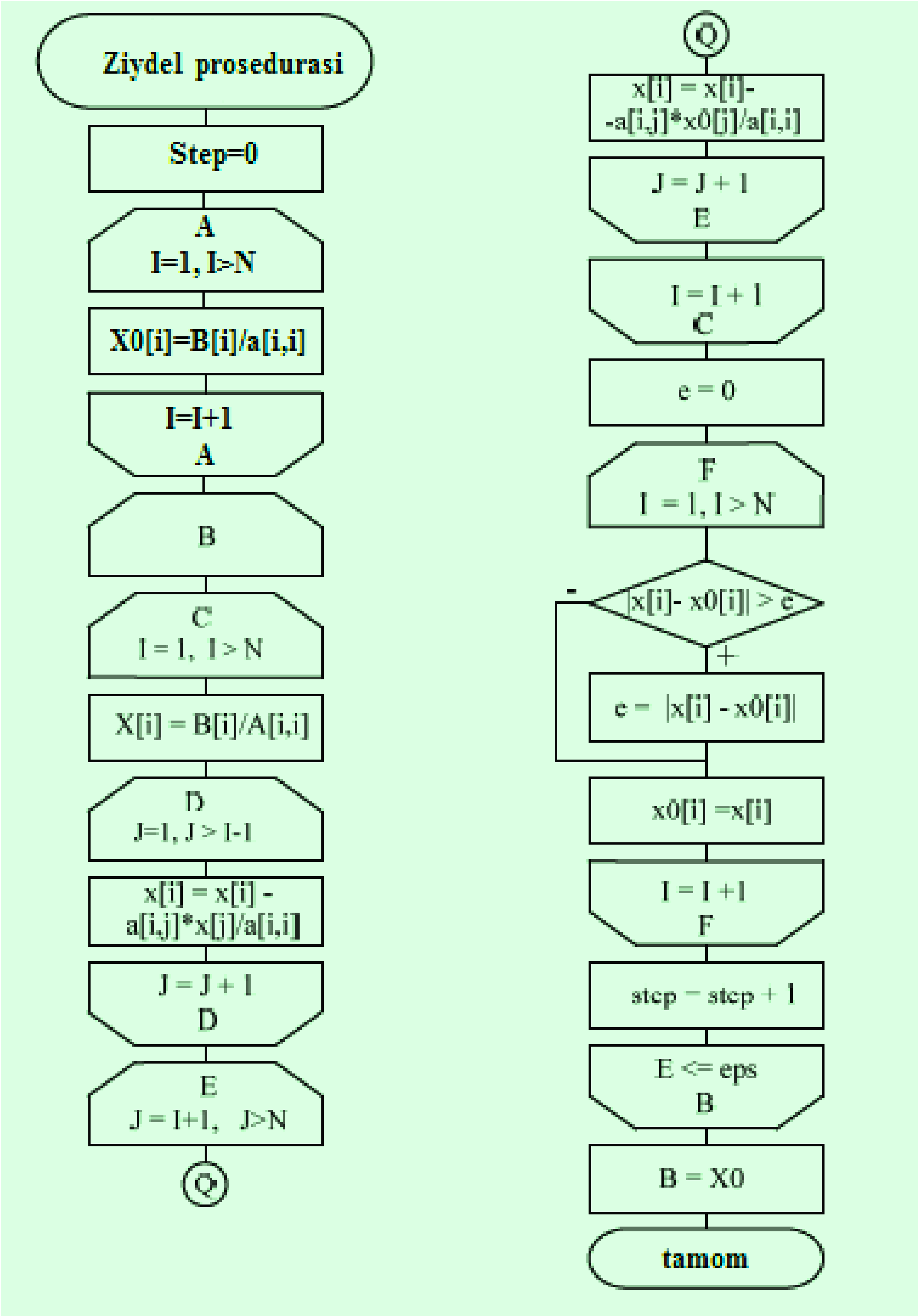
Dastur matni.

```
uses crt;
type mat=array[1..20,1..20] of real; vector=array[1..30] of real;
var  ag,temp,a,y,b,z,a2: mat;  temp2:array[1..20,1..20] of integer;
 i,p,q,j,k,n,nn,t: integer;  aa,aas,d,m,x,r,bg,x3,x2: vector;
 ii: integer;    m2,s2,max,l,s,f: real;    h: integer;
 begin     write('Tenglamalar soni N=');    readln(n);
  for i:=1 to n do     begin
   for j:=1 to n do
      begin  write('a[',i,'][',j,']=');   readln(a[i,j]);   end;
     write('b[',i,']=');    readln(a[i,j+1]);   end;
 For i:=1 To n do
For j:=1 To n+1 do a2[i,j]:=a[i,j]/a[i,i];
For i:=1 To n do x[i]:= a2[i,n+1];
repeat
For i:=1 To n do   begin   s:=a2[i,n+1];
For j:=1 To n do  begin
If j<i Then s:=s-a2[i,j]*x3[j]; If j>i Then s:=s-a2[i,j]*x[j]; end;
x2[i]:=x3[i]; x3[i]:=s;  end;  f:=0;
For i:=1 To n do    begin  If Abs(x3[i]-x2[i])>0.00001 Then f:=1;
x[i]:=x3[i];  end;
until f<>1;
writeln('Ziydel ildizlari'); for k:=1 to n do
writeln('X[',k,']=', x[k]:5:5);
for t:=1 to n do   begin   l:=a[t,t];  for j:=1 to n+1 do
a[t,j]:=a[t,j]/l;
for i:=t+1 to n do  begin
l:=a[i,t];
for j:=1 to n+1 do a[i,j]:=a[i,j]-a[t,j]*l;
 end;    end;  end.
```

Dastur natijasi :

```
CRT - программа завершена
a[3][1]=2
a[3][2]=1
a[3][3]=-20
a[3][4]=2
a[3][5]=-3
b[3]=-19
a[4][1]=0
a[4][2]=1
a[4][3]=-1
a[4][4]=10
a[4][5]=-5
b[4]=10
a[5][1]=1
a[5][2]=2
a[5][3]=-1
a[5][4]=-2
a[5][5]=-20
b[5]=-32
Ziydel ildizlari
X[1]=1.01625
X[2]=0.99756
X[3]=1.04988
X[4]=1.76597
X[5]=1.52148
```

Ziydel usuliga tuzilgan algoritm blok – sxemasi:



Quyidagi tenglamalar sistemasini Zeydel usulida ε=0,000001 aniqlikda yeching. (N-talabaning tartib raqami)

$$\begin{cases} 12{,}1x_1 - 0{,}87x_2 + 4{,}1x_4 + x_5 - 2{,}8x_6 + 1{,}1x_7 - x_9 + 0{,}24x_{10} = N - 1 \\ 0{,}8x_1 - 19{,}8x_2 + 0{,}7x_3 - 1{,}1x_4 + 0{,}7x_5 - 0{,}92x_6 + x_7 - 1.5x_8 - 2x_9 + 1{,}35x_{10} = 3 - N \\ 1{,}2x_1 + 1{,}5x_2 + 20{,}7x_3 - 1.9x_4 - 0{,}6x_5 - 0{,}22x_6 + 0.5x_8 - 1{,}4x_9 + 2{,}75x_{10} = N + 2 \\ 1{,}43x_1 - 1{,}18x_2 + 0{,}2x_3 - 22x_4 - 1{,}27x_5 - 0{,}66x_6 - x_7 + 1{,}15x_8 - x_9 + 1{,}4x_{10} = -5{,}1 \\ 3{,}1x_1 + 1{,}71x_3 - 1{,}02x_4 + 33{,}1x_5 - 1{,}85x_6 - 3x_7 - 1{,}9x_8 - 4{,}5x_9 + 0{,}16x_{10} = 8{,}97 \\ 0{,}42x_1 - 1{,}4x_2 - 0{,}17x_3 - 1{,}83x_4 + 3{,}6x_5 - 27{,}92x_6 - 0{,}8x_7 + 2.5x_8 - x_9 + 5x_{10} = -9{,}88 \\ -1{,}44x_2 + 3{,}7x_3 - x_4 + x_5 - 0{,}99x_6 + 18{,}78x_7 + 5{,}5x_8 + 0{,}2x_9 + 0{,}75x_{10} = 10 \\ 7{,}8x_1 - 10{,}8x_2 + 0{,}1x_3 + 4{,}1x_4 + 0{,}3x_5 - 0{,}72x_6 + 4{,}8x_7 - 39{,}5x_8 - 0{,}87x_9 + 3x_{10} = -N \\ 5{,}2x_1 + 1{,}01x_2 + 0{,}42x_3 - x_4 + 1{,}17x_5 + 0{,}2x_6 + 2x_7 - 1{,}5x_8 + 19{,}2x_9 + 0{,}63x_{10} = 63{,}3 \\ 0{,}09x_1 - 1{,}26x_2 + 0{,}15x_3 - 1{,}01x_4 + 0{,}19x_5 - 2{,}52x_6 + 0{,}47x_7 - 0{,}41x_8 + 0{,}2x_9 + 19{,}5x_{10} = 20 \end{cases}$$

Nazorat savollari

1. Iterasiya usullari haqida umumiy tushuncha bering?
2. Iterasion jarayon yaqinlashish shartini tushuuntirib bering?
3. Oddiy iterasiya usuli algortmi haqida tushuntiring?
4. Chiziqli tenglamalar tizimini yechish usullari haqida ma'lumot bering.
5. Zeydel usulini qo'llab chiziqli tenglamalar tizimini yechish algoritmini tuzing.
6. Zeydel usulini qo'llab chiziqli tenglamalar tizimini yechish dasturini tuzing.

5-Laboratoriya ishi

Nochiziqli tenglamalar sistemasini yechishning oddiy iteratsiya usuli va uning algoritmi.

Ishning maqsadi : Nochiziqli tenglamalar sistemasini yechishning oddiy iteratsiya usuli va uning algoritmi bilan tanishish.

Nazariy qism

Shu paytgacha biz faqat chiziqi tenglamalar tizimini yechish usullari bilan tanishdik, endi tenglamalar tizimi chiziqli bo'lmagan hol ustida to'xtalamiz. Soddalik uchun ikki noma`lumli ikkita chizimi bo'lmagan tizimni oddiy ityeratsiya usuli bilan yechishga to'xtalamiz. Bunday tizim quyidagicha yoziladi:

$$\begin{cases} f(x, y) = 0 \\ \varphi(x, y) = 0 \end{cases} \qquad (5.1)$$

Faraz qilaylik boshlangich x, y yaqinlashishlar berilgan bo`lsin. Berilgan tizimni quyidagicha yozamiz:

$$\begin{cases} x = F(x, y) \\ y = \Phi(x, y) \end{cases} \qquad (5.2)$$

hamda bu tizimning o’ng tomonidagi x va y lar o`rniga boshlangich yaqinlashish x va y larni qo’yib, birinchi yaqinlashishni aniqlaymiz:

$$\begin{cases} x_1 = F(x_0, y_0) \\ y_1 = \Phi(x_0, y_0) \end{cases} \qquad (5.3)$$

Xuddi shuningdek ikkinchi yaqinlashishni aniqlaymiz:

$$\begin{cases} x_2 = F(x_1, y_1) \\ y_2 = \Phi(x_1, y_1) \end{cases} \qquad (5.4)$$

va umuman

$$\begin{cases} x_n = F(x_{n-1}, y_{n-1}), \\ y_n = \Phi(x_{n-1}, y_{n-1}), \end{cases} \qquad (5.5)$$

Agarda *(x, y)* va F(x, y*)* funktsiyalar uzluksiz, hamda $x_1, x_2, \ldots, x_n, \ldots$ va $y_1, y_2, \ldots, y_n, \ldots$ ketma-ketliklar yaqinlashuvchi bo`lsa, u holda ularning limitlari berilgan tenglamaning yechimi bo’ladi.

Teorema. x va $\bar{y}$ (9.1) tizimning aniq yechimlari, $a < \bar{x} < b$, $c < \bar{y} < d$ bo`lib, *x=a, x=b, y=c* va *y=d* to`g’ri chiziqlar bilan chegaralangan to`g’ri to’rtburchak ichida boshqa yechimlar yo`q bo`lsa, u holda ko`rsatilgan to’g’ri to’rtburchakda quyidagi

$$\left|\frac{\partial F}{\partial x}\right| \le r_1, \qquad \left|\frac{\partial F}{\partial y}\right| \le q_1, \qquad \left|\frac{\partial \Phi}{\partial x}\right| \le r_2, \qquad \left|\frac{\partial \Phi}{\partial y}\right| \le q_2$$

($r_1 + r_2 \le M < 1$ va $q_1 + q_2 \le M < 1$) tengsizliklar bajarilsa, iteratsion jarayon yaqinlashuvchi bo’ladi va boshlangich yaqinlashish *x, y* sifatida to`g’ri to’rtburchakning ixtiyoriy nuqtasini olish mumkin.

Teoremaning isbotini keltirib o’tirmaymiz.

Endi nochiziqli sistemani ildizlarini iterasiya usulida hisoblash algoritmini ABCPascal algoritmk tilida tuzilgan dasturiga to’xtalamiz.

Dastur matni

```
program dastur_far; uses crt;
label 1,2,3,4,5;
var x,y,x0,y0,eps,ax,ay,bx,by:real;
i,n:integer;
function f(z:real; w:real):real;
   begin
            f:= (z*sqr(z)+w*sqr(w))/6+0.5;
  end;
                     function r(k:real; t:real):real;
                        begin
                             r:=(k*sqr(k)+t*sqr(t))/6+0.33;
                        end;
 begin
   4: write('x='); readln(x);
            write('y='); readln(y);
                          write('n='); readln(n);
                                  write('eps='); readln(eps);
                  ax:=sqr(x)/2; ay:=sqr(y)/2;
                   bx:=sqr(x)/2; by:=-sqr(y)/2;
                   if abs(ax+bx)<1 then goto 5 else goto 4;
  5: begin if abs(ay+by)<1 then goto 2 else goto 4;end;
  2: x0:= f(x,y); y0:=r(x,y);
                     for i:=1 to n do begin
  if abs(x0-f(x0,y0))<eps then goto 3 else x0:=f(x0,y0);
   3: if abs(y0-r(x0,y0))<eps then goto 1 else y0:=r(x0,y0); end;
   1:writeln('x0=', x0:5:4);
   writeln('y0=',y0:5:4);
   write('takrorlanishlar soni=', i);
   end.
```

1-Misol. Chiziqsiz tenglamalar sistemasini oddiy iterasiya usuli bilan yeching.

$$\begin{cases} f(x,y) = x^3 + y^3 - 6x + 3 = 0 \\ \varphi(x,y) = x^3 - y^3 - 6y + 2 = 0 \end{cases}$$

Yechish: Berilgan tizimni quyidagi ko`rinishda yozib olamiz:

$$x = \frac{x^3 + y^3}{6} + \frac{1}{2} = F(x,y)$$

$$y = \frac{x^3 - y^3}{6} + \frac{1}{3} = \Phi(x,y)$$

$0 \le x \le 1$, $0 \le y \le 1$ kvadratni qaraymiz. Agarda x_0, y_0 nuqta shu kvadratga tegishli bo'lsa, u holda $0 < F(x_0, y_0) < 1$ va $0 < F(x_0, y_0) < 1$ bo`ladi. (x_0, y_0) boshlangich yaqinlashish qanday tanlanishidan qat'iy nazar (x_k, y_k) yaqinlashishlar kvadratga tegishli bo'ladi, chunki

$$0 < \left(x_0^3 + y_0^3\right) / \ 6 < \frac{1}{3}$$

$$-1/6 < \left(x_0^3 - y_0^3\right) / \ 6 < \frac{1}{6}$$

Bundan tashqari (x_k, y_k) nuqtalar $\frac{1}{2} < x < \frac{5}{6}, \quad \frac{1}{6} < y < \frac{1}{2}$ kvadratga tegishli. Bu kvadrat nuqtalari uchun:

$$\left|\frac{\partial F}{\partial x}\right| + \left|\frac{\partial F}{\partial y}\right| = \frac{x^2}{2} + \frac{y^2}{2} < \frac{\frac{25}{36} + \frac{1}{4}}{2} = \frac{34}{72} < 1$$

$$\left|\frac{\partial \Phi}{\partial x}\right| + \left|\frac{\partial \Phi}{\partial y}\right| = \frac{x^2}{2} + \left|\frac{y^2}{2}\right| < \frac{34}{72} < 1$$

bajariladi. Demak, ko`rsatilgan kvadratda tizim yagona yechimga ega va uni iteratsion usulda aniqlash mumkin.

$x_0 = \frac{1}{2}$ va $y_0 = \frac{1}{2}$ deb olamiz, u holda

$$x_1 = \frac{1}{2} + \frac{\frac{1}{8} + \frac{1}{8}}{6} = 0{,}542, \qquad y_1 = \frac{1}{3} + \frac{\frac{1}{8} + \frac{1}{8}}{6} = 0{,}333$$

$$x_2 = \frac{1}{2} + \frac{0{,}542^3 + 0{,}333^3}{6} = 0{,}533$$

$$y_2 = \frac{1}{3} + \frac{0{,}542^3 - 0{,}333^3}{6} = 0{,}354$$

$$x_3 = \frac{1}{2} + \frac{0{,}533^3 + 0{,}354^3}{6} = 0{,}533$$

$$y_3 = \frac{1}{3} + \frac{0{,}533^3 - 0{,}354^3}{6} = 0{,}351$$

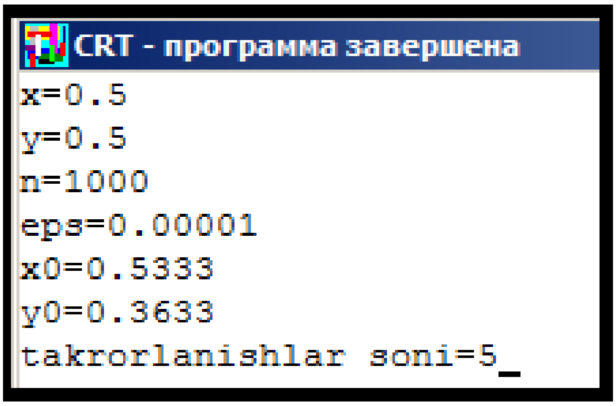
$$x_4 = \frac{1}{2} + \frac{0{,}533^3 + 0{,}351^3}{6} = 0{,}532$$

$$y_4 = \frac{1}{3} + \frac{0{,}533^3 - 0{,}351^3}{6} = 0{,}351$$

Bu yerda $q_1 = q_2 = 34/72 < 0{,}5$ bo`lgani sababli birinchi uchta o'nlik raqamlarning mos tushganligi kerakli aniqlikdagi yechimni topish imkoniyatini beradi. Shunday qilib quyndagi yechimga ega bo'ldik.

$$x = 0{,}532; \qquad y = 0{,}351$$

Natija:



Chiziqsiz tenglamalar sistemasida yaqinlashish sharti sifatida Yakobi matrisasisni tuzish o'zining soddaligi bilan ajralib turadi. Shuning uchun ko'p hillarda berilgan sistemani yechishda boshlang'ich ildizlarni berish uchun Yakobi matrisasidan foydalaniladi. Quyidagi misolda berilgan sistema uchun yechish algortmi umumiy holda va dastur matni hususiy holda keltirildi.

2-Misol. Chiziqli bo'lmagan tenglamalar sistemasini oddiy iterasiya usulida yeching.

$$\begin{cases} \cos(x+0{,}5)+y=0{,}8 \\ \sin y-2x=1{,}6 \end{cases}$$

Yechish. Sistemaga ekvivalent sistemani tuzamiz:

$$\begin{cases} y=0{,}8-\cos(x+0{,}5) \\ x=0{,}5\sin y-0{,}8 \end{cases}$$

Hususiy hosilalarni hisoblaymiz:

$$\beta_x^{'}=\sin(x+0{,}5),\quad \beta_y^{'}=0 \ \text{va}\ \alpha_x^{'}=0,\quad \alpha_y^{'}=0{,}5\cos y$$

U holda Yakobi matrisasi ko'rinishi quyidagicha bo'ladi:

$$\varphi^{'}(x,y)=\begin{pmatrix} \sin(x+0{,}5) & 0 \\ 0 & 0{,}5\cos y \end{pmatrix}$$

Boshlangich yaqinlashish sifatida (0; 0) nuqtani olamiz.

Berilgan sistemani ABC Pascal tizimida yechish dasturini tuzamiz va ildizni hisoblaymiz (Hisoblash jarayoni quyida keltirilgan).

Mustaqil bajarish uchun tajriba ishi variantlari

Chiziqli bo'lmagan tenglamalar sistemalarini 0,00001 aniqlikda oddiy iterasiya usulida yeching.

(har bir talaba tartib raqamiga mos variyantni tanlaydi!!)

$\begin{cases} 2x^2-xy-5x+1=0 \\ x+31g-y^2=0 \end{cases}$	$\begin{cases} 2x^2-xy-y^2+2y+6=0 \\ y-x-1=0 \end{cases}$
$\begin{cases} \sin x-y-1{,}32=0 \\ \cos y-x+0{,}85=0 \end{cases}$	$\begin{cases} \sin(x+1-y)=0 \\ 2x+\cos y=2 \end{cases}$

$\begin{cases} \cos(x-1)+y=0{,}8 \\ x-\cos y=2 \end{cases}$	$\begin{cases} \cos(\frac{1}{3}(x-y))-2y=0 \\ \sin(\frac{1}{3}(x+y))-2x=0 \end{cases}$
$\begin{cases} x-e^{-y}=0 \\ y-e^{x}=0 \end{cases}$	$\begin{cases} tg(x-1)-y=0 \\ x-tgy=0 \end{cases}$
$\begin{cases} \cos(x+1)+2y+1=0 \\ \sin(y+1)+2x+2=0 \end{cases}$	$\begin{cases} x^2+2y-3=0 \\ y^2x^2-3x^2-4y^2+6=0 \end{cases}$
$\begin{cases} \alpha x^2-y^2-1=0 \\ xy^3-y-4=0 \end{cases}$ $\alpha=1+0.5K$ (K=0.1,.., 5)	$\begin{cases} \alpha x^2-y^2-1=0 \\ xy^3-y-3=0 \end{cases}$ $\alpha=1+1.5;$ 2; 2.5; 3
$\begin{cases} tg(xy+K)=x^2 \\ \alpha x^2+y^2=1 \end{cases}$ $\alpha=1+0.5K$ (K=0.1,… 5)	$\begin{cases} \sin(x+Ky)-xy=-1 \\ \frac{x^2}{a}-y^2=\frac{3}{4} \end{cases}$ $a=0.75(0.25)2.00;\quad K=-1.0(-0.3)-2.2$
$\begin{cases} \cos(ky+x^2)+x^2+y^2=1{,}6 \\ 1{,}5(x+0{,}8)^2-\frac{(y-0{,}1)^2}{a^2}=1{,}4 \end{cases}$ $a=0{,}6(0{,}2)1{,}6;\ k=0{,}6(0{,}6)1{,}0$	$\begin{cases} tg(ax+y)-axy=0{,}3 \\ x^2+y^2=k \end{cases}$ $a=-1{,}2(0{,}2)-0{,}2,;\ k=1{,}3(0{,}2)2{,}2$
$\begin{cases} \sin x+2y=2 \\ \cos(y+1)+x=0{,}7 \end{cases}$	$\begin{cases} tg(xy+0{,}1)=x^2 \\ x^2+2y^2=1 \end{cases}$
$\begin{cases} \cos x+y=1{,}5 \\ 2x-\sin(y-0{,}5)=1 \end{cases}$	$\begin{cases} \sin(x+y)-1{,}2x=0{,}2 \\ x^2+y^2=1 \end{cases}$
$\begin{cases} \sin(x+0{,}5)-y=1 \\ \cos(y-2)+x=0 \end{cases}$	$\begin{cases} \{tg(xy+0{,}3)=x^2 \\ 0{,}9x^2+2y^2=1 \end{cases}$
$\begin{cases} \cos(x+0{,}5)+y=0{,}8 \\ \sin(y-2x=1{,}6 \end{cases}$	$\begin{cases} \sin(x+y)-1{,}3x=0 \\ x^2+y^2=1 \end{cases}$
$\begin{cases} \sin(x-1)=1{,}3-y \\ x-\sin(y+1)=0{,}8 \end{cases}$	$\begin{cases} tgxy=x^2 \\ 0{,}8x^2+2y^2=1 \end{cases}$
$\begin{cases} 2y-\cos(x+1)=0 \\ x++\sin y=-0{,}4 \end{cases}$	$\begin{cases} \sin(x+y)-1{,}5x=0{,}1 \\ x^2+y^2=1 \end{cases}$
$\begin{cases} \cos(x+0{,}5)-y=2 \\ \sin y-2y=1 \end{cases}$	$\begin{cases} tgxy=x^2 \\ 0{,}7x^2+2y^2=1 \end{cases}$
$\begin{cases} \sin(x+2)-y=1{,}5 \\ x+\cos(y)-2=0{,}5 \end{cases}$	$\begin{cases} \sin(x+y)-1{,}2x=0{,}1 \\ x^2+y^2=1 \end{cases}$

Dastur matni

```
uses crt; const n=2;  eps=1e-4;
type   matr=array[1..n,1..n] of real; vector=array[1..n]of real;
var a: matr; x,f,x0:vector; ITER,i,j: integer; max: real;
Function Norma(a: matr; n: integer):real;
var i,j: integer; res: real;   begin    res:=0; for i:=1 to n do
for j:=1 to n do   res:=res+A[i,j]*A[i,j]; Res:=sqrt(res); Norma:=res; end;
function func(x:vector;i:integer):real;
begin   case i of
 1:func:= (sin(x[2])-1.6)/2;
 2:func:=0.8-cos((x[1])+0.5); end; end;
function MatrJacobi(x:vector;i,j:integer):real; begin    case i of
1: case j of
1: MatrJacobi:=sin(x[1]+0.5);
2: MatrJacobi:= 0 ;  end;
2: case j of
1: MatrJacobi:=0;
2: MatrJacobi:=0.5*cos(x[2]);   end;  end;    end;
procedure vivod_matr(mat:matr;N1,N2:integer);
var i,j: integer;      begin      for i:=1 to N do begin
for j:=1 to N do write(mat[i,j]:n1:N2,' ');
writeln; end;    end;      procedure vivod_vectr(vector:vector;N1,N2:integer);
var j: integer;    begin       for j:=1 to N do
writeln('x',j,'= ',vector[j]:n1:N2);    end;
begin    clrscr;    x0[1]:=0;  x0[2]:=0; iter:=0;  repeat
 for i:=1 to n do    for j:=1 to n do
a[i,j]:= MatrJacobi (x0,i,j);  vivod_vectr(x0,3,13);
writeln('норма =',Norma(a,N));  writeln('nomer iterazii - ',iter);
writeln('================');  for i:=1 to n do X[i]:=func(x0,i);
max:=abs(X[1]-X0[1]);
for i:=2 to n do if abs(X[I]-X0[I])>max then max:=abs(X[I]-X0[I]);
X0:=X;      inc(iter); readln; until (max<eps)or(iter>20);end.
```

Natija:

```
0
x1= -0.8665196960426
x2= -0.1335042144917
норма =0.611554186843914
nomer iterazii - 9
==================
```

Nazorat savollari

1. Nochiziqli tenglama haqida tushuncha bering?
2. Nochiziqli tenglamalar sistemasini yechishning qanday usullarini bilasiz?
3. Oddiy iteratsiya usulining afzalligi va kamchiligini tushuntiring?
4. Oddiy iterasiya usuliga tuzilgan algoritm blok – sxemasini tavsiflang?

6-Laboratoriya ishi

Aniq integral qiymatini taqribiy hisoblashning to‘g‘ri to‘rtburchak, trapetsiya usullari va ularni algoritmlash

Ishning maqsadi: Aniq integral qiymatini taqribiy hisoblashning to‘g‘ri to‘rtburchak, trapetsiya usullaridan foydalanib yechish va ularni algoritmlash.

Nazariy qism

Masalaning qo‘yilishi: *[a,b]* oraliqda aniqlangan uzluksiz *f(x)* funksiya berilgan bo‘lib bizdan quyidagi

$$\int_a^b f(x)dx \qquad (6.1)$$

integralni hisoblash talab qilingan bo‘lsin. Ba’zi hollarda *f(x)* funksiyaning berilishiga qarab bu integralni aniq hisoblashimiz mumkin. Amaliy ishlarda *f(x)* funksiya shunday ko‘rinishga ega bo‘ladiki, (4.1) integralni aniq hisoblab bo‘lmaydi. Bunday hollarda (4.1) integralni berilgan aniqlikda taqribiy hisoblashga to‘g’ri keladi.

Quyida aniq integralni hisoblash uchun bir necha taqribiy usullar, ularning algoritmi va unga mos Paskal algoritmik tilida tuzilgan programmalari keltirilgan.

$[a, b]$oraliqda aniqlangan uzluksiz $f(x)$ funksiya berilgan bo‘lib bizdan quyidagi

$$I = \int_a^b f(x)dx \qquad (6.1)$$

integralni hisoblash talab qilingan bo‘lsin. Ba’zi hollarda $f(x)$ funksiyaning berilishiga qarab bu integralni aniq hisoblashimiz mumkin. Amaliy ishlarda $f(x)$ funksiya shunday ko‘rinishga ega bo‘ladiki, (6.1) integralni aniq hisoblab bo‘lmaydi. Bunday hollarda (6.1) integralni berilgan aniqlikda taqribiy hisoblashga to‘g’ri keladi.

Aniq integralni hisoblash uchun bir necha taqribiy usullar mavjud bo’lib, quyida biz bu usullar va ularning algoritmi bilan tanishamiz.

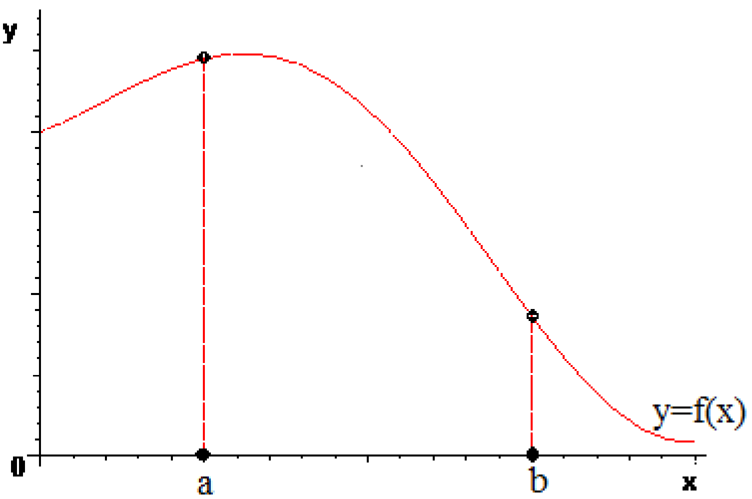
Aniq integralni hisoblashning taqribiy usullari asosida uning geometrik ma'nosi yotadi.

(6.1) ning geometrik ma'nosi shundan iboratki, (3.1) formula tekislikda yuqoridan nomanfiy $y = f(x)$ $(y \geq 0)$ funksiya egri chizig'i, pastdan absissa o'qi, o'ng va chapdan mos ravishda $x = a$, $x = b$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning yuzasini ifodalaydi (3.1-rasm).

Egri chiziqli trapesiyaning yuzasini hisoblashda ushbu g'oyaga tayaniladi.

$[a, b]$ kesmani teng n ta bo'lakga ajratiladi. Demak, qism kesmalarni ushbu $[x_i, x_{i+1}]$ $(0 \leq i \leq n-1,\ \ x_0 = a,\ \ x_n = b)$ ko'rinishda yozish mumkin. Bu qism kesmalarning har biri orasidan ξ_i ($x_i \leq \xi_i \leq x_{i+1}$) nuqta olamiz va quyidagi integral summani tuzamiz (6.2-rasm):

$$S = s_1 + s_2 + \ldots + s_n = \sum_{i=1}^{n} f(\xi_i)(x_{i+1} - x_i) \qquad (6.2)$$



6.1 – rasm. Egri chiziqli trapetsiya

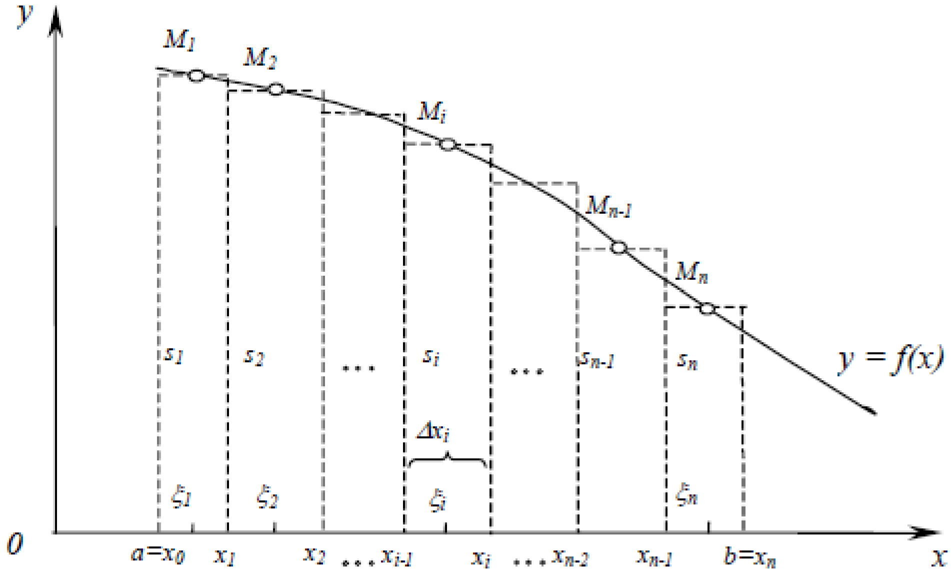
Agar bo'laklar orasidagi masofa nolga intilsa, u holda (3.2) ifodaning limiti (6.1) formula bilan aniqlangan Riman integralining qiymatini beradi, ya'ni:

$$I = \lim_{h \to 0} S \text{, bu yerda } h = x_{i+1} - x_i \qquad (6.3)$$

(6.1) integralni quyidagicha yozamiz.

$$I = \sum_{i=0}^{n-1} hf(\xi_i) + R \qquad (6.4)$$

Bu yerda (6.4) formula kvadratura, R - kvadratura xatoligi deyiladi.



6.2 – rasm. Egri chiziqli soxani to'g'ri to'rtburchaklarga ajratish

Shuningdek, (6.1) integral ostidagi funksiya murakkab bo'lganda, uni approksimatisiyalash yo'li bilan $\varphi(x)$ funksiyaga almashtirish mumkin va bunda

$$\int_a^b f(x)dx = \int_a^b \varphi(x)dx + R = S + R$$

formulaga ega bo'lamiz.

$y_i = f(x_i)$ va $h_i = \Delta x_i$ belgilashlarni kiritamiz. U holda 6.2 – rasmdagi har bir bo'lak to'g'ri to'rtburchak uchun $S_i = h_i y_i$ formula o'rinli bo'ladi.

<u>Chap tomondan to'g'ri to'rtburchak formulasi</u>. Har bir kesmadan tanlangan ξ_i nuqtani chap chegara nuqta bilan almashtirsak, ya'ni $\xi_i = x_i \quad (i = 0,1,...,n-1)$ bo'lsa, u holda

$$\int_a^b f(x)dx \approx h_1 y_0 + h_2 y_1 + ... + h_n y_{n-1}$$

formulaga ega bo'lamiz. Agar, $h_i = h = const$ deb hisoblasak,

$$\int_a^b f(x)dx \approx h\sum_{i=0}^{n-1} f(x_i) \qquad (6.5)$$

formula kelib chiqadi va (3.5) ni chapki to'g'ri to'rtburchak formulasi deyiladi.

(6.5) bo'yicha (6.1) ni hisoblash algoritm blok – sxemasi 3.3a – rasmda keltirilgan.

<u>O'ng tomondan to'g'ri to'rtburchak formulasi</u>. Har bir kesmadan tanlangan ξ_i nuqtani chap chegara nuqta bilan almashtirsak, ya'ni $\xi_i = x_{i+1}$ $(i = 0,1,...,n-1)$ bo'lsa, u holda

$$\int_a^b f(x)dx \approx h\sum_{i=1}^{n} f(x_i) \qquad (6.6)$$

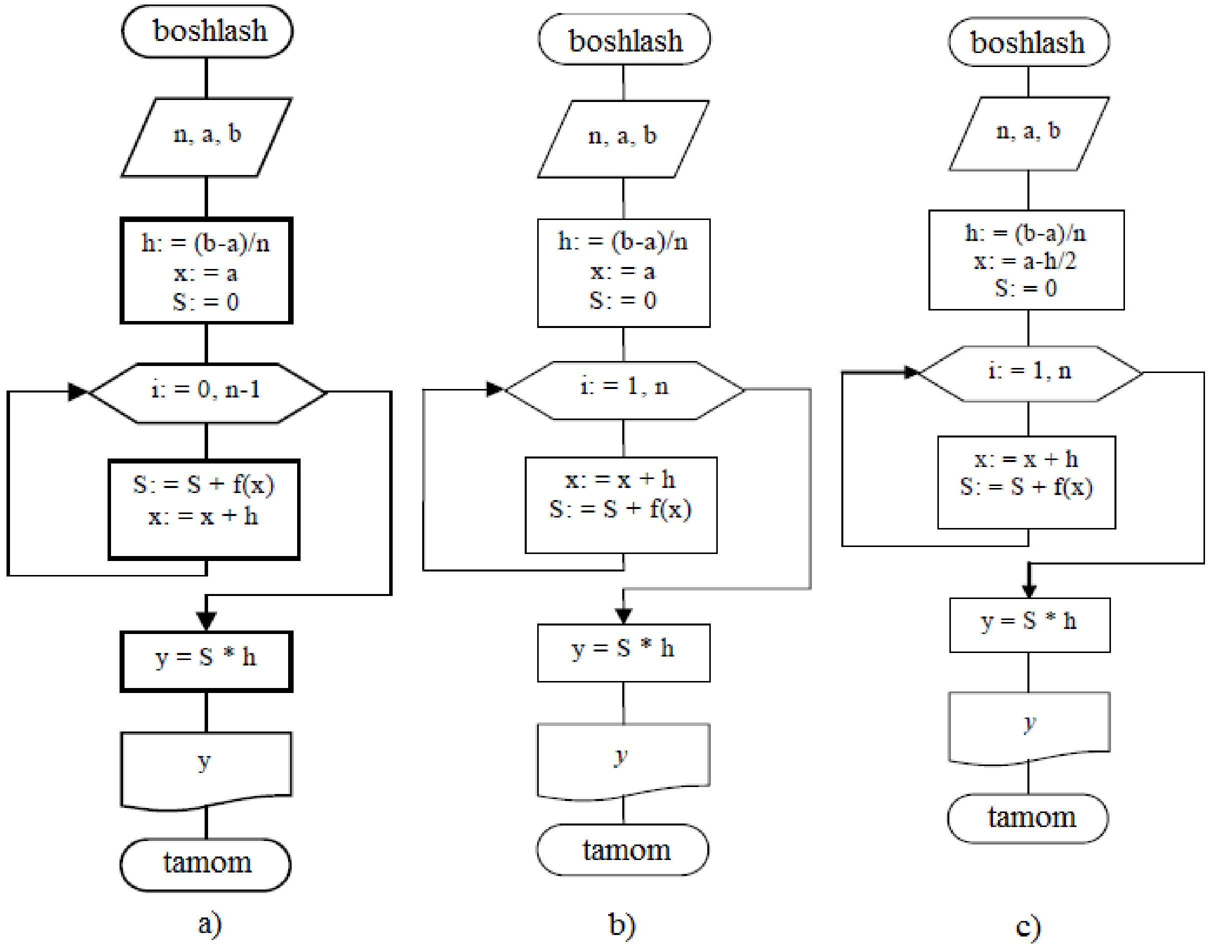
o'ngdan to'g'ri to'rtburchak formulasiga ega bo'lamiz.

(6.6) bo'yicha (6.1) ni hisoblash algoritm blok – sxemasi 3.3b – rasmda keltirilgan.

<u>O'rta to'g'ri to'rtburchak formulasi</u>. Bu usulda quyidagi formuladan foydalaniladi:

$$\int_a^b f(x)dx \approx h\sum_{i=0}^{n-1} f(x_i + h/2) \qquad (6.7)$$

Demak, to'g'ri to'rtburchak usuli bilan (3.1) ni hisoblaganda 3 ta formulaning biridan foydalanish mumkin.



6.3 – rasm. To'g'ri to'rtburchak usuliga tuzilgan algoritm blik - sxemasi

1-misol. Berilgan aniq integralning qiymatini to'g'ri to'rtburchak usulidan foydalanib hisoblang. Bunda, $n=4$, $n=10$ va $n=1000$ deb oling.

$f(x)=x(2-\sin x^2)$, $a=0$, $b=4$.

Yechish. Berilgan kesmani teng to'rtta bo'lakka ajratamiz, buning uchun $h=(b-a)/n$ formuladan qadam kattaligini hisoblaymiz: $h=(4-0)/4=1$ va $x_0=a=0$ deb qabul qilamiz.

$x_{i+1}=x_i+h$ $(i=0,1,2,3)$ formuladan $x_1=1$, $x_2=2$, $x_3=3$ va $x_4=4$ ekanligi ma'lum. Shuningdek, $f(0)=0$, $f(1)=1{,}158529$, $f(2)=5{,}513605$, $f(3)=4{,}763645$ va $f(4)=9{,}151613$ qiymatlarni hisoblaymiz. Qo'shimcha ravishda $f(0{,}5)=0{,}876298$, $f(1{,}5)=1{,}83289$, $f(2{,}5)=5{,}082948$ va so'ngida $f(3{,}5)=8{,}088918$ larga egamiz.

$\int_0^4 x(2-\sin x^2)dx$ (**) ning taqribiy qiymatini berilgan shartlar asosida hisoblaymiz.

Chapki to'g'ri to'rtburchak formulasidan foydalanamiz. Unga ko'ra

$$\int_0^4 x(2-\sin x^2) \approx I_1 = h\big(f(0)+f(1)+f(2)+f(3)\big) = 11,43578$$ kelib chiqadi.

Demak, chap formula bo'yicha $I \approx 11,43578$.

O'ng to'g'ri to'rtburchak formulasidan foydalanamiz. Unga ko'ra

$$\int_0^4 x(2-\sin x^2) \approx I_2 = h\big(f(1)+f(2)+f(3)+f(4)\big) = 20,58739$$ kelib chiqadi.

Demak, o'ng formula bo'yicha $I \approx 20,58739$.

O'rta to'g'ri to'rtburchak formulasidan foydalanamiz. Unga ko'ra

$$\int_0^4 x(2-\sin x^2) \approx I_3 = h\big(f(0,5)+f(1,5)+f(2,5)+f(3,5)\big) = 15,88105$$ o'rinli.

Demak, o'rta formula bo'yicha $I \approx 15,88105$.

Tekshiramiz: $(I_1+I_2)/2 = 16,01159$ va bu son I_3 - o'rta qiymatga yaqin ekan.

$n=10$ uchun hisoblaymiz (6.4 – rasm).

Dastur natijasiga ko'ra $[a,b]$ kesmani 10 ta bo'lakga ajratganda (**) aniq integralning taqribiy qiymati 13,7005088 ga teng ekan.

$n=1000$ uchun hisoblaymiz (6.5 – rasm).

Dastur natijasiga ko'ra $[a,b]$ kesmani 1000 ta bo'lakga ajratganda (**) aniq integralning taqribiy qiymati 15,0029083 ga teng bo'ladi.

```
program turtburchak_integral; uses crt;
var a,b,int1:real; n:integer;
function f(x:real):real;
begin  f:= x*(2-sin(sqr(x))); end;
procedure turt(a,b:real;n:integer;var int:real);
var h,s,s1,s2:real; i:integer;
begin h:=(b-a)/n; s:=0;
for i:=1 to n-1 do s:=s+f(a+i*h);
int:=h*s; end;
begin      clrscr;
write('a='); read(a);
write('b='); read(b);
write('n='); read(n);
turt(a,b,n,int1);
writeln('integral qiymatini hisoblash');
writeln('==========================');
writeln('S=',int1:10:7); end.
```

```
CRT - программа завершена
a=0
b=4
n=10
integral qiymatini hisoblash
==========================
S=13.7005088
```

6.4 – rasm. $n=4$ *uchun aniq integral qymatini hisoblash dasturi va natijasi*

```
CRT - программа завершена
a=0
b=4
n=1000
integral qiymatini hisoblash
==========================
S=15.0029083
```

6.5 – rasm. $n=1000$ *uchun aniq integral qymatini hisoblash dasturi natijasi*

2-misol. Berilgan aniq integralning qiymatini to'g'ri to'rtburchak usulidan foydalanib hisoblang. Bunda, $n=8$ va $n=8000$ deb oling.

$$\int_{0,3}^{0,9} \sqrt{5x}\ln\left(1+\frac{1}{x^2}\right)dx \qquad (6.8)$$

Yechish. (6.8) aniq integralning tqribiy qiymatini $n=8$ uchun hisoblaymiz (6.6 – rasm). Dastur natijasidan (3.8) aniq integralning $n=8$ bo’lgandagi taqribiy qiymati 6,1406367 ga teng bo’ldi.

$n=8000$ uchun hisoblaymiz (6.7 – rasm).

Berilgan (6.8) aniq integral uchun $[0{,}3; 0{,}9]$ kesmani teng sakkiz mingta bo’laklarga ajratganimizda quyidagilarni yozishimiz mumkin:

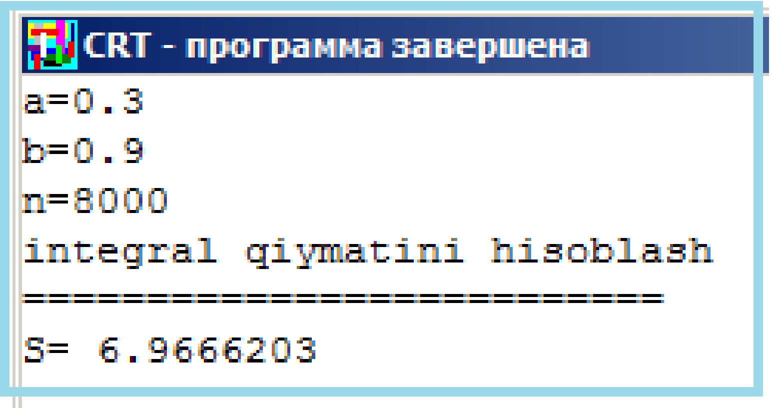
$$\int_{0,3}^{0,9} \sqrt{5x}\,\ln\left(1+\frac{1}{x^2}\right)dx \approx 6{,}9666203$$

Olingan natija (6.8) aniq integralning haqiqiy qiymatidan diyarli farq qilmaydi. Chunki, bo’laklar soni yetarlicha katta tanlandi.

```
program turtburchak_integral; uses crt;
var a,b,int1:real; n:integer;
function f(x:real):real;
begin  f:=sqr(5*x)*ln(1+1/sqr(x)); end;
procedure turt(a,b:real;n:integer;var int:real);
var h,s,s1,s2:real; i:integer;
begin h:=(b-a)/n; s:=0;
for i:=1 to n-1 do s:=s+f(a+i*h);
int:=h*s; end;
begin      clrscr;
write('a='); read(a);
write('b='); read(b);
write('n='); read(n);
turt(a,b,n,int1);
writeln('integral qiymatini hisoblash');
writeln('===========================');
writeln('S=',int1:10:7); end.
```

```
CRT - программа завершена
a=0.3
b=0.9
n=8
integral qiymatini hisoblash
===========================
S= 6.1406367
```

6.6 – rasm. $n=8$ uchun (6.8) ning qymatini hisoblash dasturi va natijasi



6.7– rasm. $n = 8000$ *uchun (6.8) ning qymatini hisoblash dasturining natijasi*

Mustaqil yechish uchun misollar.

1. Berilgan aniq integralning qiymatini to'g'ri to'rtburchak usulidan foydalanib hisoblang. Bunda, $n = 4$, $n = 40$ va $n = 4000$ deb oling, hisoblash jarayonida n ning katta qiymatlari uchun usulga tuzilgan dasturdan foydalanig.

1. $\int_{0,5}^{0,8} 5x^3 \sin(1-x)dx$ 2. $\int_{0}^{0,9} e^{1-x}(1-x^2)dx$ 3. $\int_{0,5}^{0,8} 2^x \sin x^3 dx$ 4. $\int_{1}^{2} x3^x dx$

5. $\int_{0,5}^{1} x^3 e^x dx$ 6. $\int_{0}^{1} e^{1+0,5x^2} dx$ 7. $\int_{0,5}^{1} 2^x \sin x^2 dx$ 8. $\int_{0}^{1} (x^3+1)3^x dx$ 9. $\int_{1,5}^{2} x^{x^2} dx$

10. $\int_{0,5}^{1} x^2 e^x dx$ 11. $\int_{0}^{1} e^{2+0,3x^2} dx$ 12. $\int_{0,3}^{1} 4^x \sin x^2 dx$ 13. $\int_{0}^{1} (x^3+0,1)3^x dx$

14. $\int_{1,5}^{2} 0,7x^{x^2} dx$ 15. $\int_{0,5}^{0,8} x^2(2-x^e)dx$ 16. $\int_{0,1}^{0,9} e^{1-0,5x} x^2 dx$ 17. $\int_{0}^{0,8} 2^x \cos x^3 dx$

18. $\int_{1}^{2} \sqrt{x}3^x dx$ 19. $\int_{0,5}^{0,8} x^2\sqrt{2-x}dx$ 20. $\int_{0,5}^{1} e^{2-x} x^2 dx$ 21. $\int_{0}^{0,8} 2^x \cos x^3 dx$

22. $\int_{1}^{2} \sqrt{1+x}2^x dx$ 23. $\int_{0,5}^{0,9} x^2\sqrt{3-x}dx$ 24. $\int_{0,5}^{1} e^{3-2x} x^2 dx$ 25. $\int_{0}^{0,5} 3^x \cos x^3 dx$

26. $\int_{1}^{2} \sqrt{1+2^x} dx$ 27. $\int_{0,5}^{0,9} x^{0,2}\sqrt{3^x} dx$ 28. $\int_{0,5}^{1} e^{4-3x} x^2 dx$ 29. $\int_{0}^{0,5} 5^x \cos x^3 dx$

30. $\int_{1}^{2}\sqrt{0,2+3^{x}}dx$ 31. $\int_{0,5}^{0,9}e^{0,2x}\sqrt{2^{x}}dx$ 32. $\int_{0}^{1}e^{\sin 3x}dx$ 33. $\int_{0}^{0,5}2^{1-x}5^{x}dx$

34. $\int_{1}^{2}2^{\sqrt{x}}dx$ 35. $\int_{0}^{1}e^{\sqrt{x}}dx$ 36. $\int_{0}^{1}e^{\cos 3x}dx$ 37. $\int_{0}^{0,5}x2^{1-x}dx$ 38. $\int_{0,8}^{1,1}\sin^{2}x^{3}dx$

Nazorat savollari

1. Aniq integral ta'rifini bilasizmi?
2. Aniq integralni hisoblashning qanday usullarini bilasiz?
3. Aniq integralni hisoblashda Darbu y'ig'indilari – bu nimani bildiradi?
4. Aniq integralning taqribiy qiymati deganda nimani tushunasiz?
5. Aniq integralni taqribiy hisoblashda uning geometrik ma'nosi qanday ahamiyatga ega?
6. Aniq integralni hisoblashda to'gri to'rtburchak usulining qanday formulalari mavjud?

7-Laboratoriya ishi

Aniq integral qiymatini taqribiy hisoblashning Simpson usuli va ularni algoritmlash

Ishning maqsadi: Aniq integral qiymatini taqribiy hisoblashning to‘g‘ri to‘rtburchak, trapetsiya, Simpson usullari va ularni algoritmlash

Nazariy qism

Masalaning qo‘yilishi: *[a,b]* oraliqda aniqlangan uzluksiz *f(x)* funksiya berilgan bo‘lib bizdan quyidagi

$$\int_{a}^{b} f(x)dx \qquad (7.1)$$

integralni hisoblash talab qilingan bo‘lsin. Ba'zi hollarda *f(x)* funksiyaning berilishiga qarab bu integralni aniq hisoblashimiz mumkin. Amaliy ishlarda *f(x)* funksiya shunday ko‘rinishga ega bo‘ladiki, (7.1) integralni aniq hisoblab bo‘lmaydi. Bunday hollarda (4.1) integralni berilgan aniqlikda taqribiy hisoblashga to‘g'ri keladi.

Quyida aniq integralni hisoblash uchun bir necha taqribiy usullar, ularning algoritmi va unga mos Paskal algoritmik tilida tuzilgan programmalari keltirilgan.

Simpson usuli. $h=\frac{b-a}{2n}$, $x_0=a$, $x_{2n}=b$, $y_i=f(x_i)$, $i=1,2,\ldots,n$. N- natural son bo‘lsin. Ushbu yig'indi yordamida (1) integralni taqribiy hisoblash mumkin:

$$S=\frac{h}{3}\left(y_0+y_{2n}+4\sum_{i=1}^{n}y_{2i-1}+2\sum_{i=1}^{n-1}y_{2i}\right) \qquad (7.2)$$

Xatoliklar:

$$R\le\frac{(b-a)h^4}{180}.M\ , \quad M=max|f^{IV}(z)|,\ z\in[a,b]. \qquad (7.3)$$

Misol-1. $\int_1^2\frac{1}{x^2}dx$ aniq integral qiymatini n=4 uchun taqribiy hisoblang.

Yechish : Integral belgisi ostidagi $y=\frac{1}{x^2}$ funksiya uchun, uning qiymatini [1 ;2] oraliqda n=4 da, trapetsiya formulasi bilan hisoblaymiz va uni Simpson usuli bilan aniqlangan qiymat bilan taqqoslaymiz.

$$h=\frac{b-a}{n}=\frac{2-1}{4}=0{,}25=\frac{1}{4};$$

$$S=\frac{h}{2}(y_0+y_4+2(y_1+y_2+y_3)); \quad x_0=1,\ x_1=\frac{5}{4};\ x_2=\frac{6}{3}=\frac{3}{2};\ x_3=\frac{7}{4};\ \ x_4=2;$$

$$y_0=\frac{1}{1^2}=1;\quad y_1=y(x_1)=\frac{4^2}{5^2}=\frac{16}{25}=0{,}64;\quad y_2=\left(\frac{2}{3}\right)^2=\frac{4}{9}=0{,}4444;$$

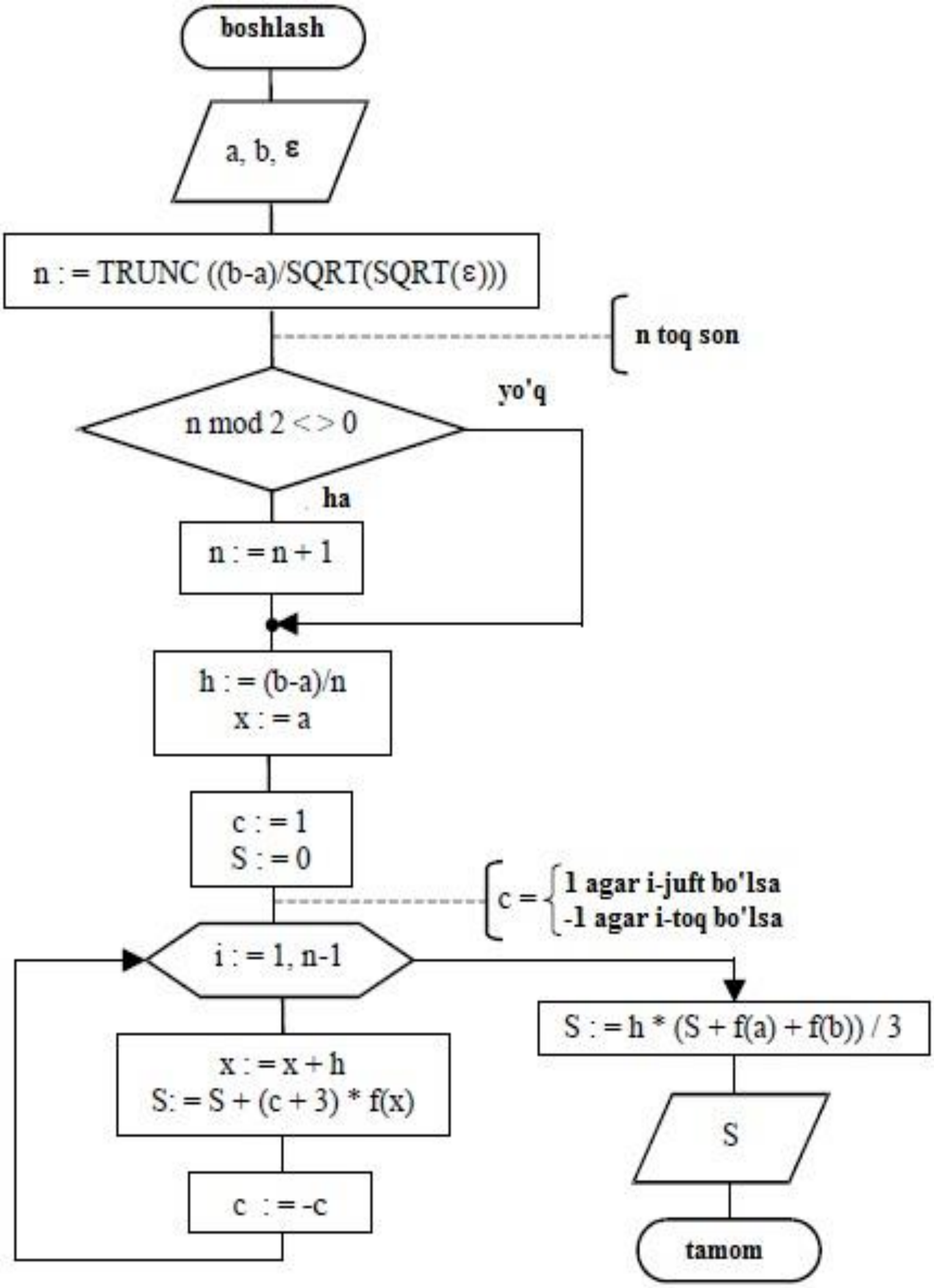
$$y_3=\left(\frac{4}{7}\right)^2=\frac{16}{49}=0{,}3265; \qquad y_4=\frac{1}{2^2}=\frac{1}{4}=0{,}25;$$

$$y_1+y_2+y_3=0{,}64+0{,}4444+0{,}3265=0{,}64+0{,}7709=1{,}4109$$

$$y_0+y_4=1+0{,}25=1{,}25$$

$$S=\frac{1}{8}(1{,}25+2{,}8218)=\frac{1}{8}3{,}0718=0{,}3839.$$

Aniq integralni Simpson usulida hisoblash algoritmi blok-sxemasi



Berilgan aniq integralni Simpson usulida hisoblash uchun Paskal tilida tuzilgan dasturning ko‘rinishi:

Dastur matni

```
program simpson; uses crt;
var a,b,int1:real; n:integer;
function f(x:real):real;
begin
        f:= 1/sqr(x);
end;
procedure simps(a,b:real;n:integer;var int:real);
var h,s,s1,s2:real; i:integer;
begin
h:=(b-a)/(2*n);
s1:=0; s2:=0;
s:=f(a)+f(b);
for i:=1 to n do s1:=s1+f(a+(2*i-1)*h);
for i:=1 to n-1 do s2:=s2+f(a+2*i*h);
int:=h*(s+4*s1+2*s2)/3;          end;
begin      clrscr;
write('a='); read(a);
write('b='); read(b);
write('n='); read(n);
simps(a,b,n,int1);
writeln('integral qiymatini hisoblash');
writeln('==========================');
writeln('S=',int1:5:6);
end.
```

Dastur natijasi

```
CRT - программа завершена
a=1
b=2
n=4
integral qiymatini hisoblash
==========================
S=0.500030
```

Misol-2. Berilgan integral tenglamaning yechimini 0,00001 aniqlikda taqribiy hisoblang.

$$\int_0^1 \frac{\sin(1+x^2)}{\ln(5+\cos x)+x^2}\,dx = x-2$$

Yechish: Integralning qiymatini S bilan belgilaymiz, u holda berilgan tenglama oddiy chiziqli ushbu

$$x = S + 2$$

tenglamadan iborat bo'ladi. S ning qiymatini 0,00001 aniqlikda hisoblash berilgan tenglamanini ildizini berilgan aniqlikda taqribiy hisoblash imkonini beradi.

Dastur matni

```
program simpson; uses crt;
var a,b,int1,y,S:real; n:integer;
function f(x:real):real;
begin
        f:= sin(1+sqr(x))/ln(5+cos(x))+sqr(x);
end;
procedure simps(a,b:real;n:integer;var int:real);
var h,s,s1,s2:real; i:integer;
begin
h:=(b-a)/(2*n);
s1:=0; s2:=0;
s:=f(a)+f(b);
for i:=1 to n do s1:=s1+f(a+(2*i-1)*h);
for i:=1 to n-1 do s2:=s2+f(a+2*i*h);
int:=h*(s+4*s1+2*s2)/3;          end;
begin       clrscr;
write('a='); read(a);
write('b='); read(b);
write('n='); read(n);
simps(a,b,n,int1);
S:=int1; y:=S+2;
writeln('Tenglama ildizini hisoblash');
writeln('==========================');
writeln('y=',y:5:5);
end.
```

Dastur natijasi

```
CRT - программа завершена
a=0
b=1
n=10000
Tenglama ildizini hisoblash
==========================
y=2.86006
```

Demak, tenglamaning 0,00001 aniqlikdagi ildizi 2,86006 ga teng ekan.

Mustaqil yechish uchun misollar.

a). Aniq integral qiymatini Simpson usulida n=100 uchun hisoblang:

1). $\int_1^2 \frac{2-\cos(2x-x^2)}{3+x^4}\sin x^2 dx$ 2). $\int_1^2 \frac{1+\cos(5x-x^2)}{1+x^4}x^2 dx$ 3). $\int_0^1 (1+x^3)e^{\cos x} dx$

4). $\int_0^1 \frac{5-\cos(x+x^2)}{3+x^4}x^3dx$ 5). $\int_1^2 \frac{4-\sin(3x-x^2)}{1+x^4}x^5dx$ 6). $\int_1^2 \frac{4x-arctg(x^4+1)}{1+x^2}dx$

b). Integral tenglamaning ildizini Simpson usulida n=100 uchun hisoblang:

1). $\int_0^1 \frac{\cos(1+x^2)}{\ln(8+\sin x)+x^2}dx = x^3-8$ 2). $\int_0^1 \frac{\ln(1+x^2)}{(8+\sin x)^2+5x^2}dx = x^3-7$

3). $\int_0^1 \frac{tgx^2}{(1{,}5+\sin x)^2+x^5}dx = x^5-11$ 4). $\int_0^1 \frac{arctg(1+x^2)}{\ln(7-\cos x)+x^2}dx = x^3-13$

5). $\int_0^1 \frac{xarctgx^2}{8+\sin x}dx = x^3-1$ 6). $\int_0^1 \frac{e^{1+x}}{\ln(2-\cos^2 x)+3x^2}dx = x^2-22$

7). $\int_1^2 \frac{ctgx^2}{(1{,}5+\sin x)^2+x^3}dx = x^2-71$ 8). $\int_0^1 \frac{arctg(1+x+x^2)}{2-\cos x+x^2}dx = x^3-53$

Nazorat savollari.

1. Aniq integralning geometrik ma’nosini ayting.
2. Taqribiy integrallash deganda nimani tushunasiz?
3. Taqribiy integrallashda to‘g’ri to‘rtburchak usuli va uning algoritmi.
4. Taqribiy integrallashda trapetsiya usuli va uning algoritmi.
5. Taqribiy integrallashda Simpson usuli va uning algoritmi.

8-Laboratoriya ishi

Oddiy differensial tenglama qiymatini taqribiy hisoblashning Eyler usuli va uni algoritmlash

Ishning maqsadi: Oddiy differensial tenglamani taqribiy hisoblashning Eyler usuli va uni algoritmlashni o’rganish.

Nazariy qism

Ma’lumki, ko’pgina muhandislik masalalarinig matematik modeli differensial tenglama uchun Koshi, chegaraviy yoki aralash masalalarni yechishga keltiriladi. Ammo bu masalalarning yechimin aniq ko’rinishda olish hamma vaqt ham mumkin bo’lavermaydi. Bu holda berilgan masalani yechish uchun taqribiy sonli yechish usullaridan foydalaniladi. Eyler usuli shunday usullardan biri hisoblanadi.

Eyler usuli. $y' = f(x,y)$ (8.1) ko’rinishdagi oddiy defferensial tenglama $[a;b]$ kesmada berilgan va uning $f(x_0)=y_0$ boshlangich shartni qanoatlantiruvchi yechimlari qiymatini hisoblash talab etilsin.

Eyler usuliga asosan $[a;b]$ kesmani N ta oraliqga ajratamiz. Bo’laklar ushbu formula bo’yicha hisoblanadi.

$x_k = a + kh$ (8.2) bu yerda $k = 0,1,2,..., N-1$ va h-oraliq qadami deyiladi va quyidagi formula yordamida hisoblanadi:

$$h = \frac{b-a}{N} \qquad (8.3)$$

Hosil bo'lgan har bir oraliqda y' hosilani taqribiy ravishda $\frac{y_k - y_{k-1}}{h}$ chekli ayirmaga almashtiramiz. Natijada noma'lum $y(x)$ funksiyaning x_k nuqtalardagi qiymatlari $y_k = y(x_k)$ ni hisoblash uchun ushbu taqribiy

$$y_k \approx y_{k-1} + hf(x_{k-1}, y_{k-1}) \qquad (8.4)$$

formulani hosil qilamiz. Bu formula va boshlang'ich shart asosida noma'lum funksiyaning $x = x_k$ nuqtalardagi qiymatlarini topish mumkin bo'ladi.

Misol. $y' = 0,5xy$ (8.5) differensial tenglamaning $y(0) = 1$ shartni qanoatlantiruvchi ildizining qiymatlarini [0;1] kesmada h=0,1 qadam bilan hisoblang.

Yechish: 8.5-tenglamaning yechimin aniq tasvirlash mumkin va uning berilgan shartni qanoatlantiruvchi yechimi $y = e^{0,25x^2}$ (8.6) funksiyadan iborat bo'ladi. Endi bu tenglamani Eyler usulida taqribiy hisoblaylik. Hisoblash jadvalini keltiramiz va bu jadvaldagi qiymatlarni taqqoslaymiz:

Aniq yechim 10 – qadamda 1,284025 va taqribiy yechim esa 1,247972 ga teng bo'lmoqda. Xatoliklarni hisoblaymiz.

Absolyut xatolik: $|1,284025 - 1,247972| = 0,036054$

Nisbiy xatoligi: $\frac{0,0361 \cdot 100}{1,284025} = 2,8\ \%$

Bundan ko'rinadiki qadamlar sonini oshirish bilan xatoliklarni kamaytirib borilsa, tenglamaning sonli taqribiy yechimlari haqiqiy ildizning son qiymatiga yaqinlashib qoladi.

K	H	X[k]	Y[k]	f(X[k],Y[k])	H*f	f(X)
0	0,1	0	1	0	0	1
1	0,1	0,1	1	0,05	0,005	1,002503
2	0,1	0,2	1,005	0,1005	0,01005	1,01005
3	0,1	0,3	1,01505	0,1522575	0,01522575	1,022755
4	0,1	0,4	1,030276	0,20605515	0,02060552	1,040811
5	0,1	0,5	1,050881	0,26272032	0,02627203	1,064494
6	0,1	0,6	1,077153	0,32314599	0,0323146	1,094174
7	0,1	0,7	1,109468	0,38831376	0,03883138	1,130319
8	0,1	0,8	1,148299	0,45931971	0,04593197	1,173511
9	0,1	0,9	1,194231	0,53740406	0,05374041	1,22446
10	0,1	1	1,247972	0,62398582	0,06239858	1,284025

Berilgan misoldagi ODT ni dasturlashtiramiz va natija olamiz.

```
program Eyler_l; uses crt;
var a,b,y0,y:real; n:integer;
function f(x,y:real):real;
begin
f:= 0.5*x*y;
end;
procedure eyler(a,b,y1:real; n:integer; y:real);
var h,x:real; i:integer;
begin h:=(b-a)/n; x:=a;
writeln('x=',x:6:2,' y=',y1:10:6);
for i:= 1 to n do begin
y:= f(x,y1) *h + y1; x:=x+h;
writeln('x~',x:6:2,' y=',y:10:6); y1:=y;
end; end;
begin clrscr;
write('a='); read(a); write('b='); read(b);
write('y0='); read(y0); write('n='); read(n);
eyler(a,b,y0,n,y); end.
```

Natija:

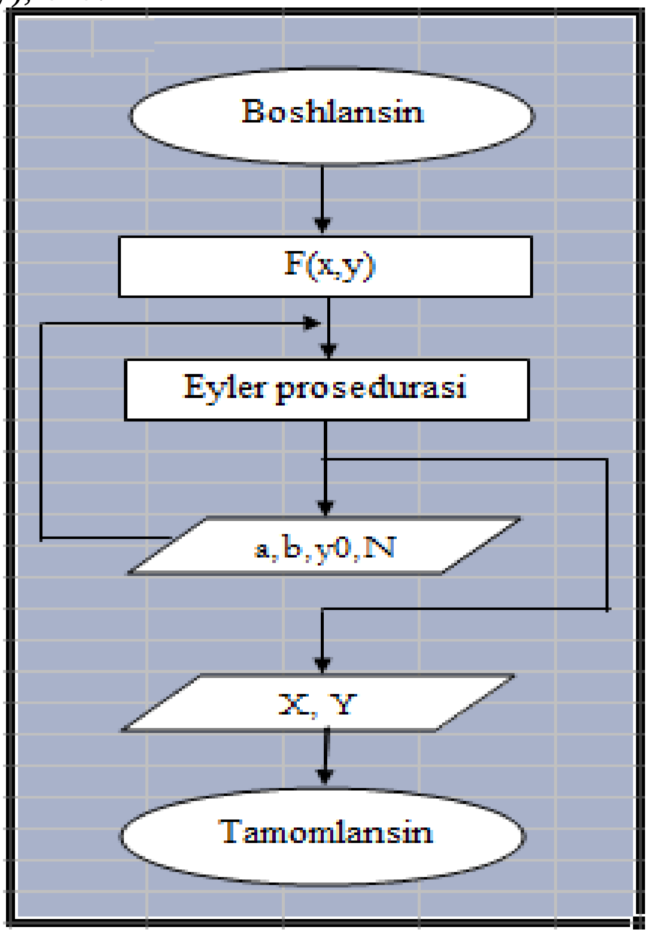
```
CRT - программа завершена
a=0
b=1
y0=1
n=10
x=   0.00  y=   1.000000
x~   0.10  y=   1.000000
x~   0.20  y=   1.005000
x~   0.30  y=   1.015050
x~   0.40  y=   1.030276
x~   0.50  y=   1.050881
x~   0.60  y=   1.077153
x~   0.70  y=   1.109468
x~   0.80  y=   1.148299
x~   0.90  y=   1.194231
x~   1.00  y=   1.247972
```

Eyler usuliga tuzilgan algoritm blok – sxemasi

Oddiy differensial tenglama uchun Eyler usuliga tuzilgan dastur matni.

```
program Eyler_l; uses crt;
 var a,b,y0,y:real; n:integer;
   function f(x,y:real):real;
     begin
          f:= ;  {funksiyaning ko'rinishi}
     end;
procedure eyler(a,b,y1:real; n:integer; y:real);
   var h,x:real; i:integer;
```

```
begin h:=(b-a)/n; x:=a;
    writeln('x=',x:6:2,' y=',y1:10:6);
        for i:= 1 to n do begin
            y:= f(x,y1) *h + y1; x:=x+h;
        writeln('x~',x:6:2,' y=',y:10:6); y1:=y; end; end;
          begin clrscr;
    write('a='); read(a); write('b='); read(b);
write('y0='); read(y0); write('n='); read(n);
eyler(a,b,y0,n,y); end.
```



Mustaqil bajarish uchun amaliy ish variantlari

1) $y'=2x-e^x+1$	$y(0)=1$	11) $y'=1-\sin x$	$y(0)=1$
2) $y'=2x\sin x+x^2\cos x$	$y(\pi)=0$	12) $y'=2x-x^2+(1-x)\ln x+1+y$	$y(1)=1$
3) $y'=e^x+y$	$y(0)=1$	13) $y'=y-e^{-x}$	$y(0)=1$
4) $y'=x+1+y$	$y(0)=1$	14) $y'=y+e^{-x}$	$y(0)=0$
5) $y'=e^x+y$	$y(0)=0$	15) $y'=2xe^{-x}-y$	$y(0)=0$
6) $y'=(x+1)^2-y$	$y(0)=1$	16) $y'=\sin x+x^2$	$y(0)=1$
7) $y'=e^x+\cos x+\sin x+y$	$y(0)=1$	17) $y'=y-e^x-1$	$y(0)=1$
8) $y'=^{x-1}-1-\ln x-x+y$	$y(1)=-1$	18) $y'=y+\cos x$	$y(0)=0$
9) $y'=1+y+(1+x)\ln x$	$y(1)=0$	19) $y'=2x\sin x-y$	$y(0)=0$
10) $y'=y\cos x$	$y(0)=1$	20) $y'=\sin x+x$	$y(0)=1$

Nazorat savollari

1. Oddiy differensial tenglamani yechish usullari haqida ma'lumot bering?

2. Oddiy differensial tenglamani yechish usullari algoritmi tuzilishi haqida?

3. Eyler usulini tushuntirib bering?

9-Laboratoriya ishi

Oddiy differensial tenglama qiymatini taqribiy hisoblashning Milna usuli va ularni algoritmlash

Ishdan maqsad: Oddiy differinsial tenglamalarni yechishning Milna usulidan foydalanib tenglama yechimini topish va uning algoritmini tuzish.

Nazariy qism

Milna usuli oddiy differensial tenglamani yechish nazariyasida muhim ahamiyatga ega bo'lgan prognoz va korreksiya usullariga bog'liq bo'lib, tenglamani yechish aniqligini oshiradi. Prognoz va korreksiya usullariga to'xtalinib o'tmagan holda, mazkur usul bilan to'g'ridan – to'g'ri tanishamiz.

Milna usuli – bu prognoz bosqichiga Milna formulasini tadbiq etish usuli hisoblanadi. Milna formulasi quyidagicha:

$$y_{i+1} = y_{i-3} + \frac{4}{3}h\left(2y_i^{'} - y_{i-1}^{'} + 2y_{i-2}^{'}\right) + O\left(h^5\right) \qquad (9.1)$$

Bu yerda $O\left(h^5\right) = \frac{28}{90}h^5 y^{(5)}$ - prognoz formulasining xatoligi. Shuningdek korreksiya bosqichiga Simpson formulasi quyidagicha bo'ladi:

$$y_{i+1} = y_{i-1} + \frac{1}{3}h\left(y_{i+1}^{'} + 4y_i^{'} + 2y_{i-1}^{'}\right) + O\left(h^5\right) \qquad (9.2)$$

Bu yerda $O\left(h^5\right) = -\frac{1}{90}h^5 y^{(5)}$ - korreksiya formulasining xatoligi.

Milna usuliga tuzilgan algoritm blok – sxemasi 9.1 – rasmda keltirilgan.

Milna usuliga tuzilgan algoritmda Runge-Kutta usuli bo'yicha aniqlanadigan qiymatlar mavjud. Shu bois bu usul to'g'risida biroz to'xtalamiz. Runge-Kutta usuli tenglamalarni yechishda ilmiy, injenerlik – texnik xarakterga ega. Koshi masalasini Runge-Kuttaning 4-tartibli aniqlik usulida yechishda to’rtta k_0, k_1, k_2 va k_3 yordamchi kattaliklar kiritiladi.

$$y_{i+1} = y_i + \frac{1}{6}\left(k_0 + 2k_1 + 2k_2 + k_3\right), \quad i = 0,1,...$$

$$k_0 = h \cdot f(x_i, y_i),$$
$$k_1 = h \cdot f(x_i + h/2, y_i + k_0/2),$$
$$k_2 = h \cdot f(x_i + h/2, y_i + k_1/2),$$
$$k_3 = h \cdot f(x_i + h, y_i + k_2).$$

Milna usuliga tuzilgan dastur matni quyida keltirilgan (Matlab):

```
function MILNA
clear all
clc
y(1)=exp(1);
a = 1; % бошлангич нуқта
b = 2; % оҳирги нуқта
N = 100; % нуқталар сони
h = (b-a)/N; % қадам катталиги
x = a:h:b;
% Рунге-Кутта методи
for i=2:(b-a)/h+1
k1=h*myfun(x(i-1),y(i-1));
k2=h*myfun(x(i-1)+h/2,y(i-1)+1/2*k1);
k3=h*myfun(x(i-1)+h/2,y(i-1)+1/2*k2);
k4=h*myfun(x(i-1)+h,y(i-1)+k3);
y(i)=y(i-1)+1/6*(k1+2*k2+2*k3+k4);
end
k1
k2
k3
k4
```

```
% ечимни текшириш
f=exp(x);
plot(x,y,'o',x,f,'-'), grid on;
legend('Рунге-Кутт усули','Ечим аниқлиги');
q=abs(max(y)-max(f));
fprintf('Ечим хаталоги = %1.10f\n',q);
function u=myfun(x,y)
u=x^2*e((x^2)/1);
```

Milna usuliga tuzilgan dastur matni:

```
#include <iomanip>
#include <math.h>
#include <iostream>
double f(double x, double y)
{ return (x*x+y*y); }
using namespace std;
void main ()
{
   int i,n=10;
   double A,B,h,E;
   h=0.05;
   E=0.001;
   double x[10],y[10],y1[10],k1,k2,k3,k4;
   x[0]=0;
   y[0]=1;
   y1[0]=f(x[0],y[0]);
/*cout<<"x[0]=";cin>>x[0];cout<<"y[0]=";cin>>y[0];cout<<"h=";cin>>h;co
ut<<"E=";cin>>E;*/
   for (i=1;i<=3;i++)
          {
```

```
        k1= h*f(x[i-1],y[i-1]);
        k2= h*f(x[i-1]+h/2,y[i-1]+k1/2);
        k3= h*f(x[i-1]+h/2,y[i-1]+k2/2);
        k4= h*f(x[i-1]+h,y[i-1]+k3);
        x[i]= x[i-1]+h;
        y[i]= y[i-1]+(k1+2*k2+2*k3+k4)/6;
        y1[i]=f(x[i],y[i]);
}
for(i=1;i<=3;i++)
{cout<<" x= "<<x[i]<<" , y= "<<y[i]<<endl;}
    i=3;
[B]do
{
    y[i+1]= y[i-3]+(4/3)*h*(2*y1[i]-y1[i-1]+2*y1[i-2]);
    x[i+1]=x[i]+h;
     B=y[i+1];
      do
    {
        A=B ;
        y1[i+1]= f(x[i+1],A);
        B=y[i-1]+h*(y1[i+1]+4*y1[i]+y1[i-1])/3;
        ++i;
    }
    while(fabs(A-B)>=E);
        y[i+1]= B;
}
while(i<=10);[/B]
     for(i=4;i<=10;i++)
    {cout<<" x = "<<x[i]<<" , y = "<<y[i]<<endl;}
```

}

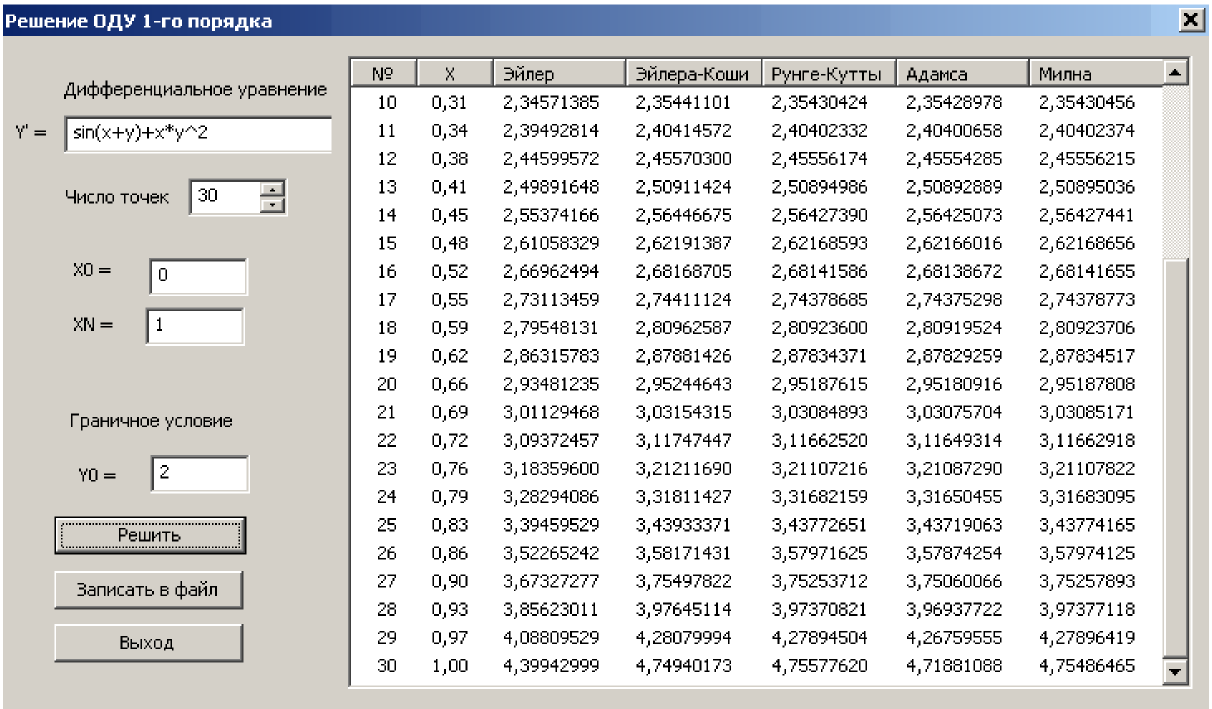
1 – misol. $y' = \sin(x+y) - 0{,}7xy^2$ oddiy differinsial tenglamaning [0,1] kesmadagi $y(0) = 2$ boshlang'ich shartni qanoatlantiruvchi taqribiy ildizini Milna usulida hisoblang va boshqa usullar bilan qiymatlarni solishtiring ($n = 10$ deb oling).

Yechish. Berilgan ODTni Milna usuliga tuzilgan dastur asosida hisoblaymiz (9.2-rasm).

Natijalardan, Eyler usulida yechim qiymati 4,0385294 ga, Eyler – Koshi usulida 4,71386059 ga, Runge – Kutta usulida 4,75372165 ga, Adams usulida 4,46868136 ga va Milna usulida 4,72304231 ga teng ekanligini ko'ramiz. Bu yerda Milna usuli bilan olingan yechim aniqligi haqida aytish qiyin, ammo tajriba o'tkazib ko'rishimiz mumkin. Nuqtalar sonini oshirish bilan yangi natijalarni olamiz. Masala shartiga ko'ra taqribiy yechim: $y \approx 4{,}72304231$.

2 – misol. $y' = \sin(x+y) - 0{,}7xy^2$ oddiy differinsial tenglamaning [0,1] kesmadagi $y(0) = 2$ boshlang'ich shartni qanoatlantiruvchi taqribiy ildizini Milna usulida hisoblang va boshqa usullar bilan qiymatlarni solishtiring ($n = 30$ deb oling).

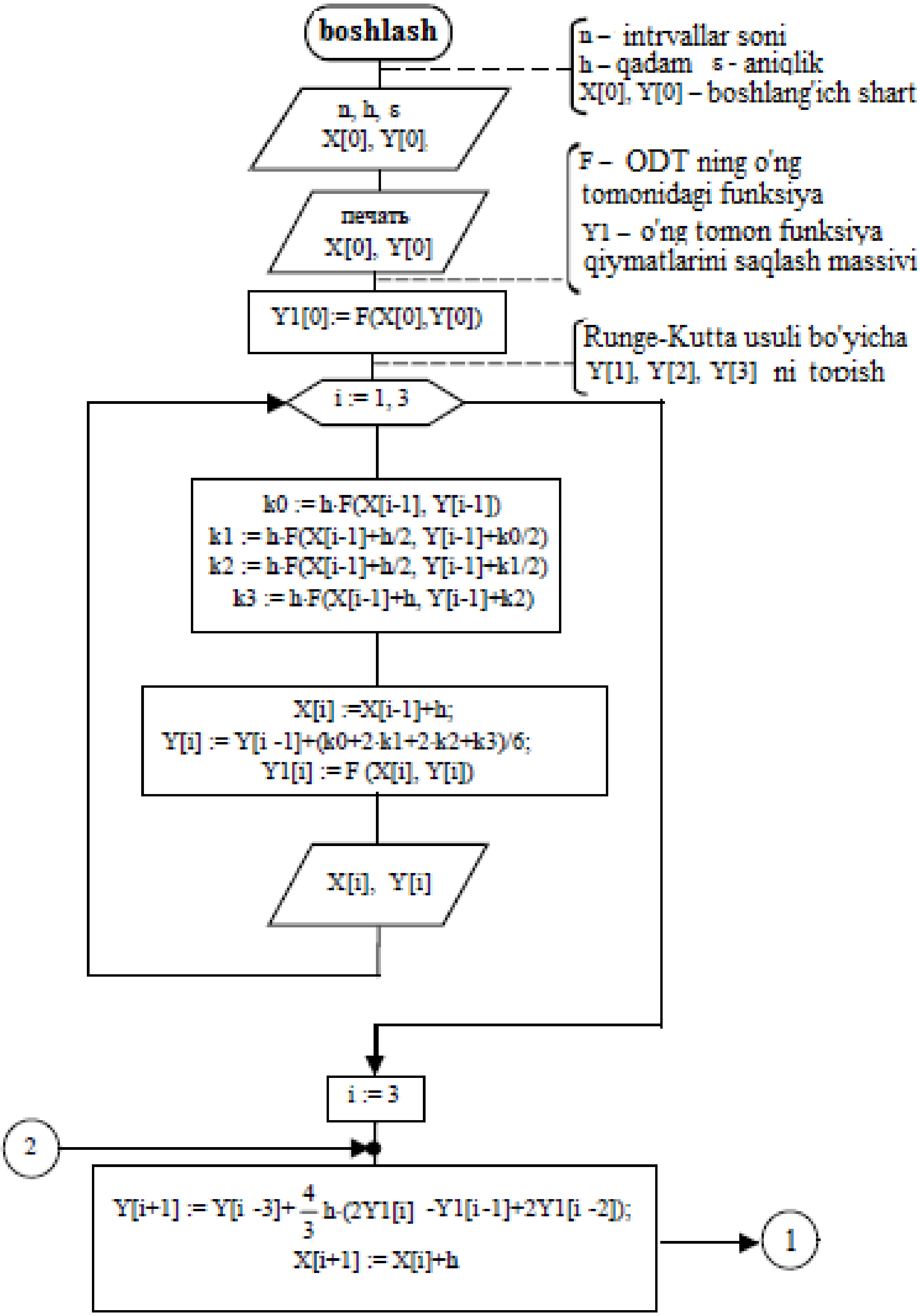
Yechish. Yuqorigi misoldagi kabi dastur natijalariga egamiz (9.2a – rasm).

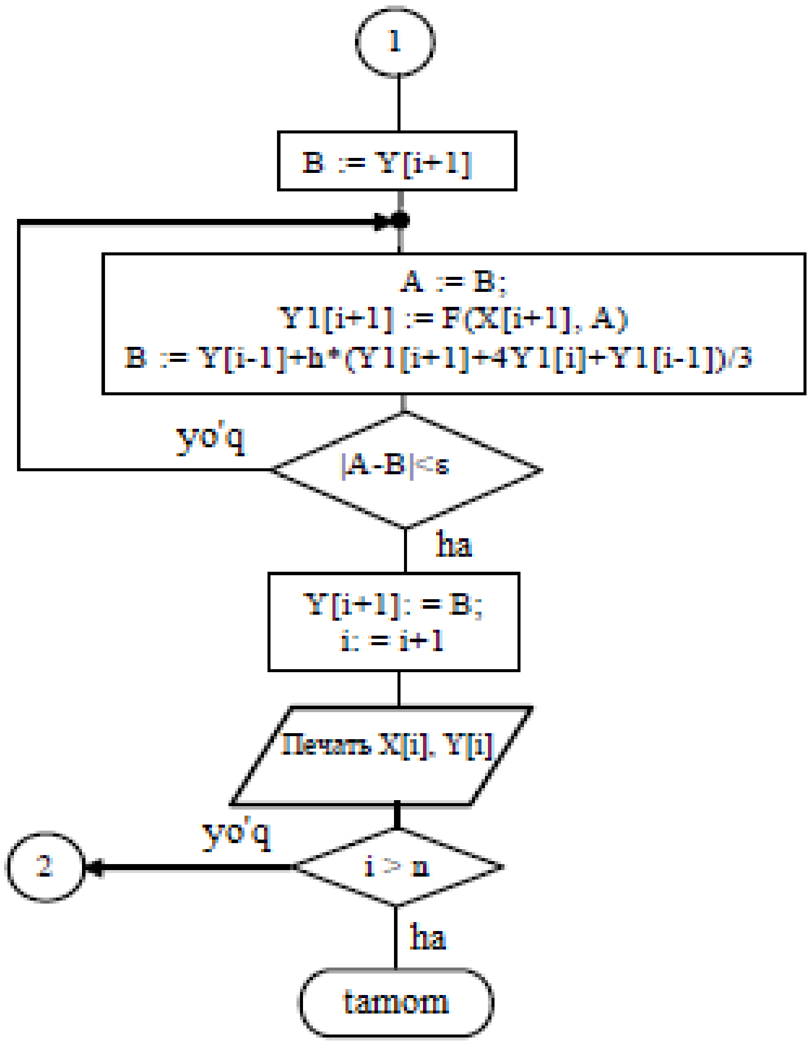


№	X	Эйлер	Эйлера-Коши	Рунге-Кутты	Адамса	Милна
10	0,31	2,34571385	2,35441101	2,35430424	2,35428978	2,35430456
11	0,34	2,39492814	2,40414572	2,40402332	2,40400658	2,40402374
12	0,38	2,44599572	2,45570300	2,45556174	2,45554285	2,45556215
13	0,41	2,49891648	2,50911424	2,50894986	2,50892889	2,50895036
14	0,45	2,55374166	2,56446675	2,56427390	2,56425073	2,56427441
15	0,48	2,61058329	2,62191387	2,62168593	2,62166016	2,62168656
16	0,52	2,66962494	2,68168705	2,68141586	2,68138672	2,68141655
17	0,55	2,73113459	2,74411124	2,74378685	2,74375298	2,74378773
18	0,59	2,79548131	2,80962587	2,80923600	2,80919524	2,80923706
19	0,62	2,86315783	2,87881426	2,87834371	2,87829259	2,87834517
20	0,66	2,93481235	2,95244643	2,95187615	2,95180916	2,95187808
21	0,69	3,01129468	3,03154315	3,03084893	3,03075704	3,03085171
22	0,72	3,09372457	3,11747447	3,11662520	3,11649314	3,11662918
23	0,76	3,18359600	3,21211690	3,21107216	3,21087290	3,21107822
24	0,79	3,28294086	3,31811427	3,31682159	3,31650455	3,31683095
25	0,83	3,39459529	3,43933371	3,43772651	3,43719063	3,43774165
26	0,86	3,52265242	3,58171431	3,57971625	3,57874254	3,57974125
27	0,90	3,67327277	3,75497822	3,75253712	3,75060066	3,75257893
28	0,93	3,85623011	3,97645114	3,97370821	3,96937722	3,97377118
29	0,97	4,08809529	4,28079994	4,27894504	4,26759555	4,27896419
30	1,00	4,39942999	4,74940173	4,75577620	4,71881088	4,75486465

9.1 – rasm. Milna usuliga tuzilgan dastur natijasi

Nuqtalar sonini oshirganimiz sababli Milna va Runge – Kutta usullari bilan olingan natijalar bir – biriga juda yaqin. Bizga ma’lumki, Runge – Kutta usuli takomillashgan usul, hamda Milna usulida Runge – Kutta usuliga murojaat etiladi.


boshlash
n – intrvallar soni
h – qadam ε - aniqlik
X[0], Y[0] – boshlang'ich shart
n, h, ε
X[0], Y[0]
печать
X[0], Y[0]
F – ODT ning o'ng
tomonidagi funksiya
Y1 – o'ng tomon funksiya
qiymatlarini saqlash massivi
Y1[0]:= F(X[0],Y[0])
Runge-Kutta usuli bo'yicha
Y[1], Y[2], Y[3] ni topish
i := 1, 3
k0 := h·F(X[i-1], Y[i-1])
k1 := h·F(X[i-1]+h/2, Y[i-1]+k0/2)
k2 := h·F(X[i-1]+h/2, Y[i-1]+k1/2)
k3 := h·F(X[i-1]+h, Y[i-1]+k2)
X[i] :=X[i-1]+h;
Y[i] := Y[i -1]+(k0+2·k1+2·k2+k3)/6;
Y1[i] := F (X[i], Y[i])
X[i], Y[i]
i := 3
2
Y[i+1] := Y[i -3]+ 4/3 h·(2Y1[i] -Y1[i-1]+2Y1[i -2]);
X[i+1] := X[i]+h
1



9.2 – rasm. Milna usuliga tuzilgan algoritm blok – sxemasi.

Demak shu asosida berilgan ODT uchun eng maqbul yaqinlashish sifatida Milna usuli bilan qo'lga kiritilgan yechimni tanlashimiz mumkin.

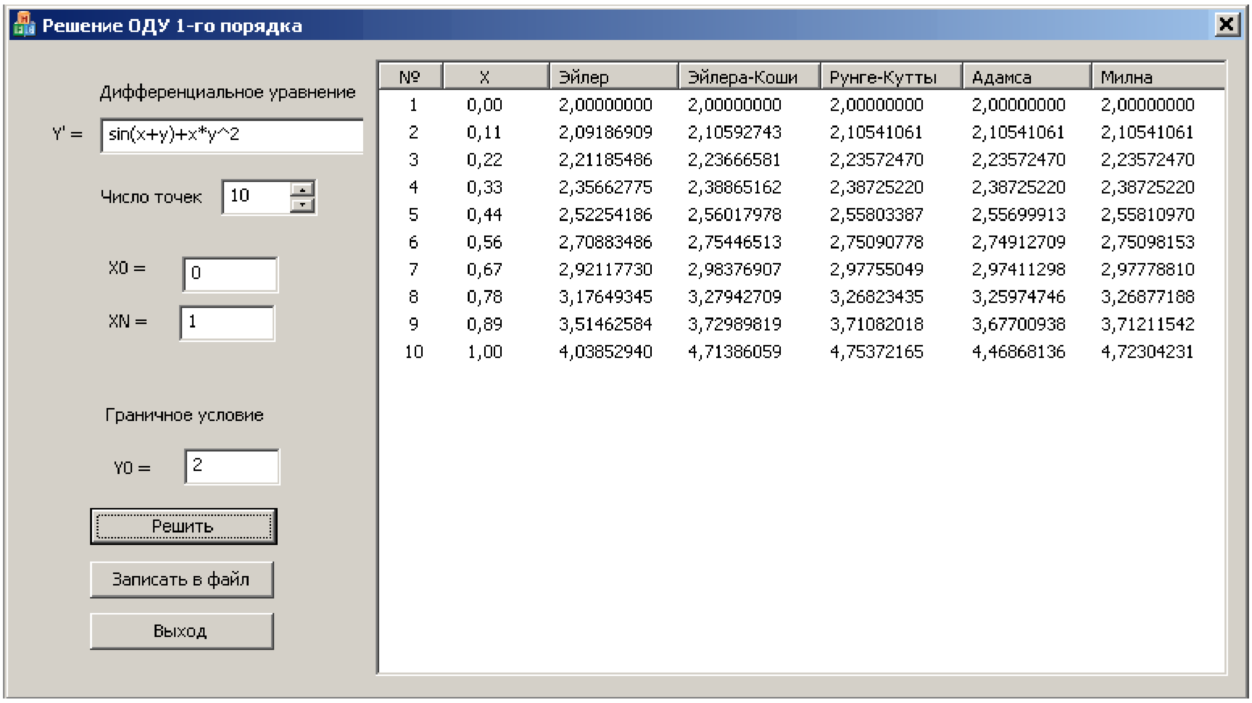
Masala shartiga ko'ra taqribiy yechim: $y \approx 4{,}75486465$.

3–misol. $y' = 3x - 0{,}5xy + 1$ oddiy differinsial tenglamaning $[0{,}1]$ kesmadagi $y(0) = 3{,}41$ boshlang'ich shartni qanoatlantiruvchi taqribiy ildizini Milna usulida hisoblang va boshqa usullar bilan qiymatlarni solishtiring ($n = 50$ deb oling).

Yechish. Yuqoridagi kabi dastur natijalariga egamiz (9.3b – rasm). Bu yerda dastur natijalardan, Eyler usulida yechim qiymati 2,88376287 ga, Eyler – Koshi usulida 2,90139150 ga, Runge – Kutta usulida 2,90120145 ga, Adams usulida

2,90120229 ga va Milna usulida 2,90120143 ga teng ekanligini ko'ramiz. Bu yerda Milna usuli bilan Runge – Kutta usulida olingan natijalar diyarli bir xil.

Masala shartiga ko'ra taqribiy yechim: $y \approx 2{,}90120143$.



№	X	Эйлер	Эйлера-Коши	Рунге-Кутты	Адамса	Милна
1	0,00	2,00000000	2,00000000	2,00000000	2,00000000	2,00000000
2	0,11	2,09186909	2,10592743	2,10541061	2,10541061	2,10541061
3	0,22	2,21185486	2,23666581	2,23572470	2,23572470	2,23572470
4	0,33	2,35662775	2,38865162	2,38725220	2,38725220	2,38725220
5	0,44	2,52254186	2,56017978	2,55803387	2,55699913	2,55810970
6	0,56	2,70883486	2,75446513	2,75090778	2,74912709	2,75098153
7	0,67	2,92117730	2,98376907	2,97755049	2,97411298	2,97778810
8	0,78	3,17649345	3,27942709	3,26823435	3,25974746	3,26877188
9	0,89	3,51462584	3,72989819	3,71082018	3,67700938	3,71211542
10	1,00	4,03852940	4,71386059	4,75372165	4,46868136	4,72304231

9.2a – rasm. Milna usuliga tuzilgan dastur natijasi

1. Berilgan oddiy differinsial tenglamalarning $[0,1]$ kesmadagi $y(0)=0{,}5$ boshlang'ich shartni qanoatlantiruvchi taqribiy ildizini Milna usulida hisoblang ($n=10$, $n=30$ deb oling).

1. $y' = 0{,}5y^2 - 0{,}4\cos(x-y)$
2. $y' = 0{,}1x^2y^2 - 4\sin(x+y)$
3. $y' = 0{,}45xy^2 + 0{,}1\cos(x^2-y)$
4. $y' = 0{,}2xy^2 - 5\cos(x-y^2)$
5. $y' = 0{,}21xy - 0{,}7\cos^2(x+y)$
6. $y' = 0{,}9\sin y^2 - 0{,}4\cos(x-y)$
7. $y' = 0{,}6\sin x^2 - 0{,}8\cos(x+y)$
8. $y' = 0{,}6\sin(x^2+y) + 0{,}1\cos(2x-y)$
9. $y' = 0{,}6\cos x^2 - 0{,}8\ln(x^2+y^2)$
10. $y' = y + x + 0{,}1\ln(1+x^2+y^2)$

Nazorat savollari

1. ODT ni sonli yechishning Milna usuli aniqligi haqida nima deyish mumkin?
2. ODT ni sonli yechishning Milna usuli afzalligi va kamchiligi nimada?
3. ODT ni sonli yechishning Milna usuliga tuzilgan algoritm blok – sxemasini tuzing. Sizningcha bu optimal variantmi?

4. Milna usulida qaysi usul g'oyalaridan foydalaniladi?
5. Milna usuli bilan yana qanday tipik masalalarni yechish mumkin?

10-Laboratoriya ishi

Oddiy differensial tenglamalar sistemasini taqribiy hisoblashning Eyler usuli va uni algoritmlash

Ishdan maqsad: Oddiy differinsial tenglamalar sistemasini taqribiy hisoblashning Eyler usulidan foydalanib yechish va algoritmlashni urganish.

Nazariy qism

Birinchi tartibli ddiy differensial tenglamalar (bundan so'ng ODTlar) ning normal sistemasi deb

$$\begin{cases} y_1' = f_1(x, y_1, y_2, ..., y_n) \\ y_2' = f_2(x, y_1, y_2, ..., y_n) \\ .... \\ y_n' = f_n(x, y_1, y_2, ..., y_n) \end{cases} \qquad (10.1)$$

ko'rinishdagi sistemaga aytiladi. Normal sistemada tenglamalar soni noma'lum funksiyalar soniga teng deb faraz qilinadi.

(10.1) sistemaning yechimi deb sistemaning hamma tenglamasini qanoatlantiradigan n ta $y_1, y_2, ..., y_n$ funksiyalar to'plamiga aytiladi.

(10.1) sistemaning xususiy yechimi deb ushbu

$$x = x_0 \text{ da } y_1 = y_{10},\ y_2 = y_{20},\ ...,\ y_n = y_{n0}$$

boshlang'ich shartlarni qanoatlantiradigan yechimga aytiladi.

Oddiy differensial tenglamalr sistemasini yechishning Eyler usuliga to'xtalamiz.

Bu usul ODTni yechishda Eyler usuli kabi xarakterga ega bo'lib, quyidagi hisoblash jarayonini hosil qiladi.

$$\begin{aligned} &y_1(x+h) = y_1(x) + h \cdot f_1(x, y_1(x), y_2(x), ..., y_n(x)), \\ &y_2(x+h) = y_2(x) + h \cdot f_2(x, y_1(x), y_2(x), ..., y_n(x)), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &y_n(x+h) = y_n(x) + h \cdot f_n(x, y_1(x), y_2(x), ..., y_n(x)). \end{aligned} \qquad (10.2)$$

ODTlar sistemasini Eyler usulida yechish algoritm blok – sxemasi 10.1-rasmda keltirilgan.

1-misol. Berilgan ODTlar sistemasini $x=0$ da $y_1(0)=1$ va $y_2(0)=0,2$ boshlang'ich shartlarni qanoatlantiruvchi taqribiy yechimini Eyler usulida hisoblang ($h=0,01$, $k=10$ deb oling).

$$\begin{cases} y_1' = xy+1 \\ y_2' = -\sin(x-y) \end{cases} \qquad (10.3)$$

Yechish. Dastlab $f(x,y)=xy+1$ va $g(x,y)=\sin(y-x)$ belgilashlarni kiritamiz. Endi bizda hisoblash uchun barchasi tayyor bo'ldi. Dastur tuzamiz va undan natija olamiz (10.1a va 10.1b-rasmlar).

Dastur natijsiga ko'ra $y_1 \approx 1{,}1047941244$ va $y_2 \approx 0{,}2162210986$ yechimlar kelib chiqadi.

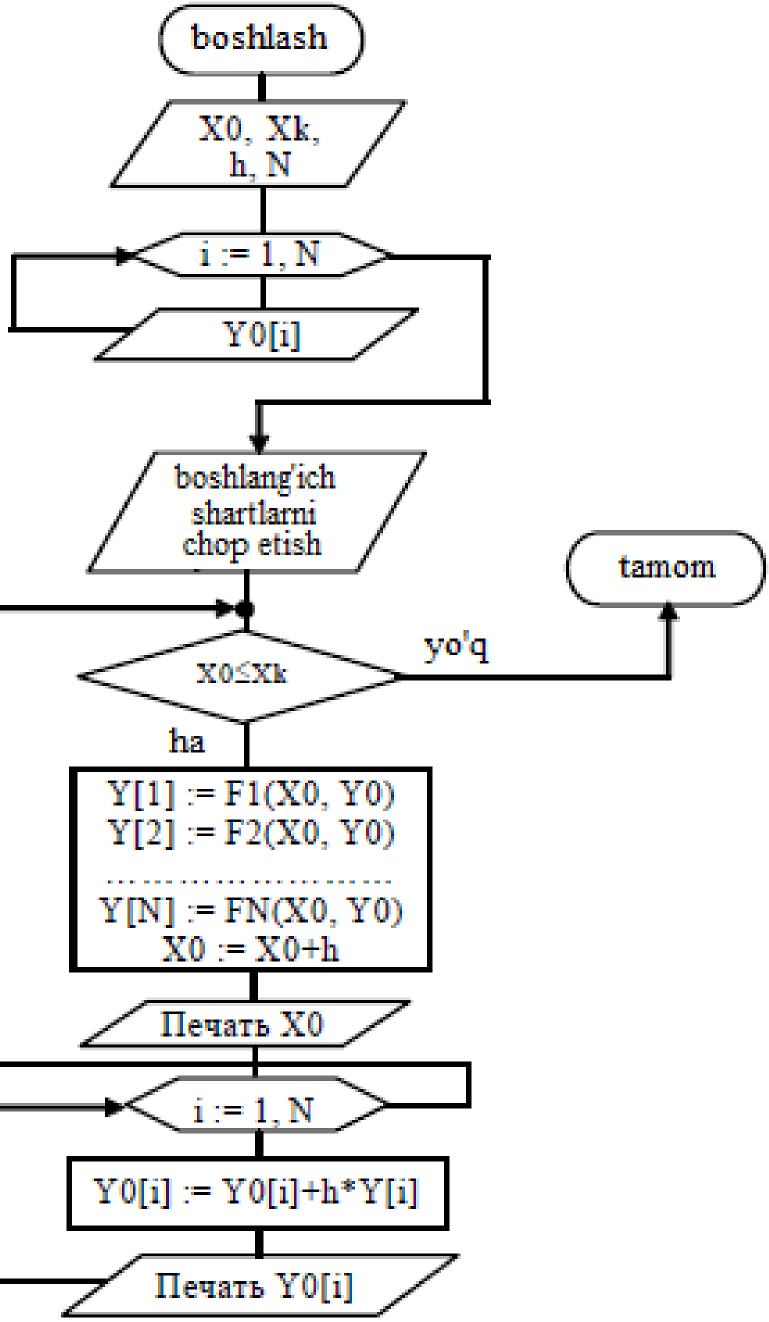
2-misol. Berilgan ODTlar sistemasini $x=1$ da $y_1(0)=1$ va $y_2(0)=2{,}31$ boshlang'ich shartlarni qanoatlantiruvchi taqribiy yechimini Eyler usulida hisoblang ($h=0,01$, $k=10$ deb oling).

$$\begin{cases} y_1' = xy+0{,}1x^2 \\ y_2' = x-\sqrt{x^2+0{,}2y^2} \end{cases} \qquad (10.4)$$

Yechish. Yuqoridagi kabi belgilash kiritamiz va dastur natijasini olamiz (10.1-rasm). Dastur natijasidan (4.14) sistemaning taqribiy yechimlarini quyidagicha yozamiz:

$$y_1 \approx 1{,}1210006560 \text{ va } y_2 \approx 2{,}2681385093.$$

Biz yuqorida faqat y_1 ning ishtirokidagi sistemani ko'rib chiqdik, endi y_1 va y_2 funksiyalar to'liq qatnashgan sistemani yechishni 42-misolda ko'rib chiqamiz.



10.1 - rasm. ODTlar sistemasini Eyler usulida yechish algoritm blok - sxemasi

```
program dig_sistema_Eyl; uses crt;
label 1;
const n=2;
var x0,xk,y1,y2,y10,y20,h:real; i,k,j:integer;
function f(x:real; y:real):real;
begin f:=x*y+1; end;
function g(x:real;y:real):real;
begin g:=-sin(x-y); end;
begin
write('x0='); read(x0); write('y10='); read(y10);
write('y20='); read(y20); write('h='); read(h);
write('k='); read(k); write('x0=',x0:5:3);
write('         f(x0)=',y10:5:3);
writeln(' g(x0)=',y20:5:3);
writeln('---------------------------------------');
         xk:=x0+h*k; y1:=y10; y2:=y20;
for i:=1to k do begin
 if x0>xk then goto 1 else begin
 y1:=y1+h*f(x0,y1); y2:=y2+h*g(x0,y2); x0:=x0+h; end;
write('x=',x0:3:2); write('     y1=',y1:5:10);
writeln('      y2=',y2:5:10); end;
1: END.
```

10.1a-rasm. (10.4) sistemani hisoblash dastur matni

```
CRT - программа завершена
x0=0
y10=1
y20=0.2
h=0.01
k=10
x0=0.000        f(x0)=1.000 g(x0)=0.200
---------------------------------------
x=0.01     y1=1.0100000000      y2=0.2019866933
x=0.02     y1=1.0201010000      y2=0.2038947879
x=0.03     y1=1.0303050202      y2=0.2057233886
x=0.04     y1=1.0406141117      y2=0.2074715929
x=0.05     y1=1.0510303574      y2=0.2091384914
x=0.06     y1=1.0615558725      y2=0.2107231679
x=0.07     y1=1.0721928061      y2=0.2122246993
x=0.08     y1=1.0829433410      y2=0.2136421563
x=0.09     y1=1.0938096957      y2=0.2149746033
x=0.10     y1=1.1047941244      y2=0.2162210986
```

10.1b-rasm. (10.4) sistemani hisoblash dastuining natijasi

```
CRT - программа завершена
x0=1
y10=1
y20=2.31
h=0.01
k=10
x0=1.000        f(x0)=1.000  g(x0)=2.310
----------------------------------------
x=1.01     y1=1.0110000000      y2=2.3056221698
x=1.02     y1=1.0222312000      y2=2.3012886023
x=1.03     y1=1.0336983582      y2=2.2969986546
x=1.04     y1=1.0454063513      y2=2.2927516942
x=1.05     y1=1.0573601774      y2=2.2885470988
x=1.06     y1=1.0695649592      y2=2.2843842561
x=1.07     y1=1.0820259478      y2=2.2802625637
x=1.08     y1=1.0947485255      y2=2.2761814288
x=1.09     y1=1.1077382095      y2=2.2721402686
x=1.10     y1=1.1210006560      y2=2.2681385093
```

10.2-rasm. (10.5) sistemani hisoblash dastuining natijasi

Mustaqil yechish uchun misollar.

1. Berilgan ODTlar sistemasini $x=0$ da $y_1(0)=1$ va $y_2(0)=0,2$ boshlang'ich shartlarni qanoatlantiruvchi taqribiy yechimini Eyler usulida hisoblang ($h=0,01$, $k=10$ deb oling).

1. $\begin{cases} y_1' = x + 0{,}1y \\ y_2' = xy + 0{,}9\sin y \end{cases}$ 2. $\begin{cases} y_1' = x^2 y + 0{,}3 \\ y_2' = y + 0{,}5\sin xy \end{cases}$ 3. $\begin{cases} y_1' = xy^2 + 0{,}8 \\ y_2' = x + 0{,}41\cos xy \end{cases}$

2. Berilgan ODTlar sistemasini $x=1$ da $y_1(0)=1$ va $y_2(0)=2,31$ boshlang'ich shartlarni qanoatlantiruvchi taqribiy yechimini Eyler usulida hisoblang ($h=0,01$, $k=10$ deb oling).

1. $\begin{cases} y_1' = x\sqrt{y} + 0{,}3x \\ y_2' = y + 0{,}1\sin(x-y) \end{cases}$ 2. $\begin{cases} y_1' = x^2 y^2 + 0{,}3x \\ y_2' = 0{,}23\sin^2 xy + 1 \end{cases}$ 3. $\begin{cases} y_1' = y^2 + 0{,}8x \\ y_2' = x^2 + 0{,}56\cos y \end{cases}$

3. Boshlang'ich shartlari $x_0=0$, $y_0=1$, $z_0=0$ bo'lgan differensial tenglamalar sistemasini yeching. ($h=0,01$, $k=10$ deb oling).

1. $\begin{cases} y' = x - 2y \\ z' = x(y - z) \end{cases}$ 2. $\begin{cases} y' = yx - 2y^2 \\ z' = xy - z \end{cases}$ 3. $\begin{cases} y' = x^2 - 2zy^2 \\ z' = xyz \end{cases}$ 4. $\begin{cases} y' = zx^2 - 2y^2 \\ z' = x - yz \end{cases}$

Nazorat savollari

1. ODT lar sistemasi deb qanday sistemaga aytiladi?
2. ODT lar sistemasining yechimi deb nimaga aytiladi?
3. ODT lar sistemasini sonli yechishning qanday usullarini bilasiz?
4. ODT lar normal sistemasi deganda nimani tushunasiz?
5. ODT lar sistemasining yechishning Eyler usuli algoritm blok – sxemasini tasvirlab berolasizmi?

11-Laboratoriya ishi

Nuqtali approksimatsiyalash. Lagranj ko'phadini hisoblash usuli va uning algoritmi

Ishning maqsadi: Tajriba natijalarini Nyuton va Lagranj usuli bilan interpolyatsiyalash usulini algoritmlash va hisoblash dasturini tuzishni o'rganish bo'yicha amaliy ko'nikmani hosil qilish.

Nazariy qism

Lagranj interpolyatsion formulasi. Algebraik interpolyatsion masalaning qo'yilishi qo'yidagichadir. Darajasi n dan yuqori bo'lmagan shunday ko'phad qurilsinki, u berilgan $(n+1)$ ta $x_0, x_1, ..., x_n$ nuqtalarda berilgan $f(x_0), f(x_1), ..., f(x_n)$ qiymatlarni qabul qilsin. Bu masalani geometrik ta'riflashham mumkin: darajasi n dan ortmayidigan shunday $P(x)$ ko'phad qurilsinki, uning grafigi berilgan $(n+1)$ ta $M_k(x_k, f(x_k)), (k = 0,1,2,...,n)$ nuqtalardan o'tsin.

Demak, c_m koeffitsientlarni shunday aniqlash kerakki, $P(x) = c_0 + c_1x + c_2x^2 + ... + c_nx^n$ ko'phad uchun ushbu $P(x_k) = f(x_k), (k = 0,1,...,n)$ tengliklar bajarilsin. Bu tengliklarni ochib yozsak, $c_m, (m = 0,1,...,n)$ larga nisbatan $(n+1)$ noma'lumli $(n+1)$ ta tenglamalar sistemasi hosil bo'ladi:

$$\begin{cases} c_0 + c_1x_0^1 + c_2x_0^2 + ... + c_nx_0^n = f(x_0) \\ c_0 + c_1x_1^1 + c_2x_1^2 + ... + c_nx_1^n = f(x_1) \\ .... \\ c_0 + c_1x_n^1 + c_2x_n^2 + ... + c_nx_n^n = f(x_n) \end{cases}$$

Bu sistemani yechib, c_m larni topib o'rniga qo'yisak, $P(x)$ ko'phad aniqlanadi. Biz $P(x)$ ning oshkor ko'rinishini topish uchun boshqacha yo'l tutamiz, avvalo fundamental ko'phadlar deb ataluvchi $Q_{nj}(x)$ larni, ya'ni

$$Q_{nj}(x) = \delta_i^j = \begin{cases} 0, i \neq j \\ 1, i = j \end{cases}$$

shartlarni qanoatlantiradigan n-darajali ko'phadlarni quramiz. U holda

$$L_n(x) = \sum_{j=0}^{n} f(x_j) Q_{nj}(x) \quad (11.1)$$

izlanayotgan interpolyatsion ko'phad bo'ladi. Haqiqatan ham, barcha $i = 0,1,2,...,n$ lar uchun

$$L_n(x_i) = \sum_{j=0}^{n} f(x_j) Q_{nj}(x_i) = \sum_{j=0}^{n} f(x_j) \delta_i^j = f(x_i)$$

va ikkinchi tomondan $L_n(x)$ ***n***-darajali ko'phaddir.

Endi $Q_{nj}(x)$ ning oshkor ko'rinishini topamiz, $j \neq i$ bo'lganda $Q_{nj}(x_i) = 0$, shuning uchun ham $Q_{nj}(x)$ ko'phad $j \neq i$ bo'lganda $x - x_i$ ga bo'linadi. Shunday qilib, n –darajali ko'phadning n ta bo'luvchilari bizga ma'lum, bundan esa

$$Q_{nj}(x) = C \prod_{i \neq j} (x - x_i)$$

kelib chiqadi. Noma'lum kupayituvchi C ni esa

$$Q_{nj}(x_j) = C \prod_{i \neq j} (x_j - x_i) = 1$$

shartdan topamiz; natijada:

$$Q_{nj}(x) = \prod_{i \neq j} \frac{x - x_i}{x_j - x_i}$$

bu ifodani (6.1) ga qo'yib, kerakli ko'phadni aniqlayimiz:

$$L_n(x) = \sum_{j=0}^{n} f(x_j) \prod_{i \neq j} \frac{x - x_i}{x_j - x_i}$$

bu ko'phad ***Lagranj*** interpolyatsion ko'phadi deyiladi.

Bu formulaning xususiy hollarini qarayimiz: n=1 bo'lganda Lagranj ko'phadi ikki nuqtadan o'tuvchi to'g'ri chiziq formulasini beradi:

$$L_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

Agar n=2 bo'lsa, u vaqtda kvadratik interpolyatsion ko'phadga ega bo'lamiz, bu ko'phad uchta nuqtadan o'tuvchi va vertikal o'qqa ega bo'lgan parabolani aniqlayidi:

$$L_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2).$$

Misol. 0, 1, 2 nuqtalarda mos ravishda 1, 2, 5 qiymatlarni qabul qiluvchi kvadratik ko'phad qurilsin.

Yechish. Bu qiymatlarni oxirgi formulaga qo'yamiz:

$$L_2(x)=\frac{(x-1)(x-2)}{(0-1)(0-2)}*1+\frac{(x-0)(x-2)}{(1-0)(1-2)}*2+\frac{(x-0)(x-1)}{(2-0)(2-1)}*5=x^2+1.$$

Topilgan interpolyatsion ko'phad xatoligini qo'yidagi formuladan foydalanib baholayimiz:

$|R_n(x)|\le\frac{M_{n+1}}{(n+1)!}|x-x_0||x-x_1|...|x-x_n|$ bu yerda $M_{n+1}=\max\limits_{a\le x\le b}|f^{n+1}(x)|$. Bu yerda $f^{n+1}(x)$ - funksiyadan olingan n+1 tartibli hosilasini bildiradi.

Misol. $f(x)=\ln(x)$ funksiya uchun x=2, 3, 4 tugun nuqtalari yordamida Lagranj ko'phadini tuzish. $\bar{x}_0=2{,}5$ nuqtada funksiya qiymatini hisoblash va interpolyatsion polinom xatoligini baholash. Interpolyatsiya tugunlaridagi funksiya qiymatlari qo'yidagi jadvalda keltirilgan:

x	2	3	4
$f(x)$	0,6931	1,0986	1,3863

Yechish. Bu yerda n=2 bo'lganligidan Lagranjning

$$L_2(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2)$$

interpolyatsion polinomiga ko'ra

$$L_2(x)=\frac{(x-3)(x-4)}{(2-3)(2-4)}0{,}6931+\frac{(x-2)(x-4)}{(3-2)(3-4)}1{,}0986+\frac{(x-2)(x-3)}{(4-2)(4-3)}1{,}3863$$

yoki

$$L_2(x)=-0{,}4713+0{,}7x-0{,}0589x^2.$$

Ushbu polinomdan foydalanib, $x=\bar{x}_0$ nuqtadagi qiymatni hisoblayimiz: $L_2(x)=0{,}910575$.

Endi tuzilgan interpolyatsion polinom xatoligini qo'yidagi formula yordamida baholayimiz:

$|R_n(x)|\le\frac{M_{n+1}}{(n+1)!}|x-x_0||x-x_1|...|x-x_n|$ bu yerda $M_{n+1}=\max\limits_{a\le x\le b}|f^{n+1}(x)|$.

Hosilalarni hisoblayimiz:

$$f'(x)=(\ln(x))'=\frac{1}{x},\ f''(x)=-\frac{1}{x^2},\ f'''(x)=\frac{2}{x^3};M_3=\max f'''(x)=\frac{1}{4}$$

demak, $M_3=\frac{1}{4}$, u holda $|R_2(2{,}5)|\le\frac{1}{4}*\frac{0{,}5*0{,}5*1{,}5}{3!}=0{,}0156$.

Umuman olganda xatolik 0.0156 dan katta bo'lmayidi. Bunga iqror bo'lish uchun olingan natijalarni taqqoslash yetarli:

$$\ln(2{,}5) = L_2(2{,}5) = 0{,}9163 - 0{,}9106 = 0{,}0057 .$$

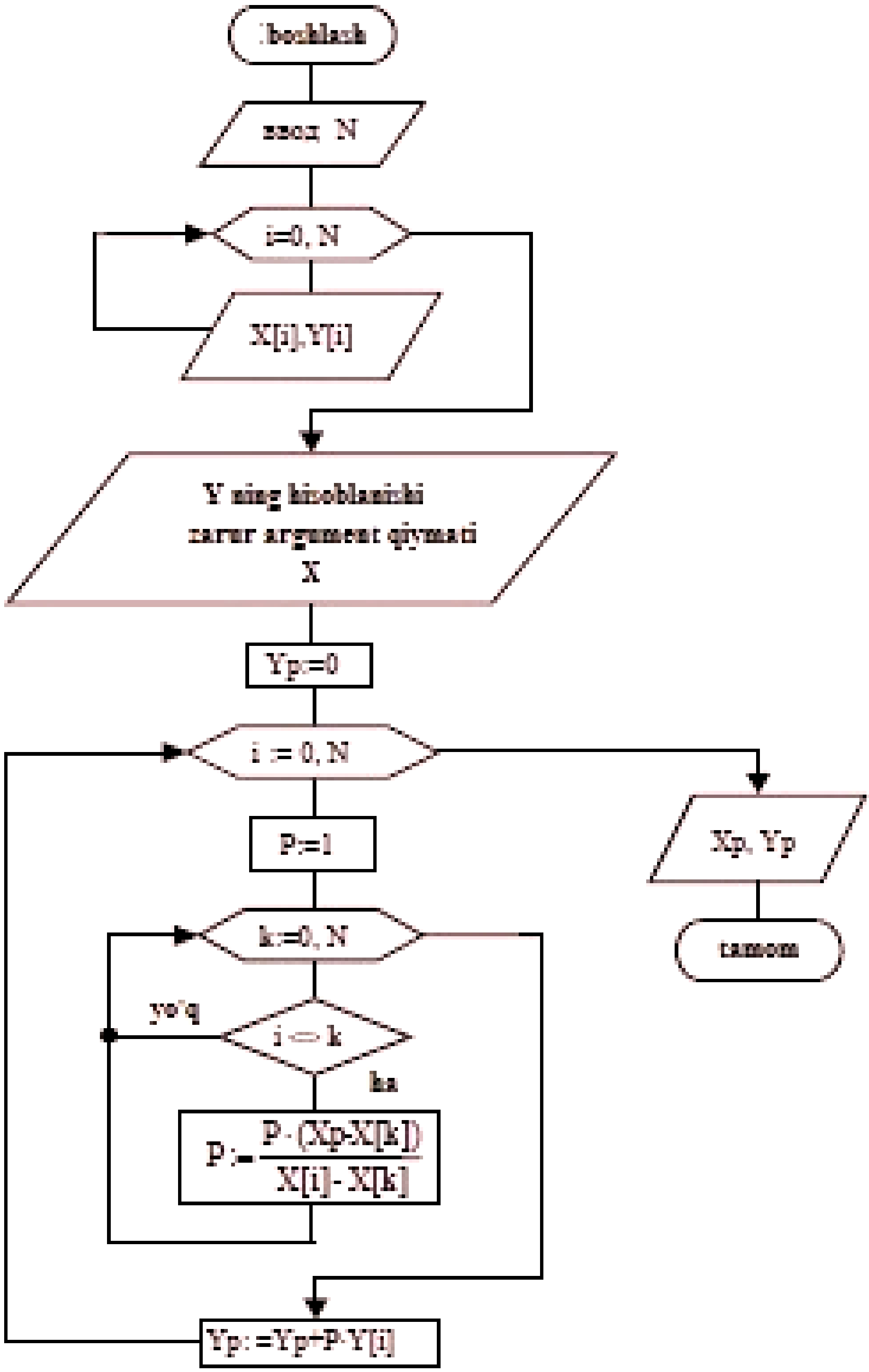
Dastur matni

```
  program kuphad_jag; uses crt;
const  n=3 ;
type vek=array[0..n] of real;
var I,j:integer;  x,y:vek; x1,y1:real;
procedure lagran(x:real; k:integer; px,py:vek; var lag:real);
var s1:real;
begin    lag:=0;
for i:=0 to k  do  begin s1:=1.0;
for j:=0 to i-1 do s1:= s1*(x-px[j])/(px[i]-px[j]);
for j:=i+1 to k do s1:=s1*(x-px[j])/(px[i]-px[j]);
lag:=lag+s1*py[i]
end; end;
begin
writeln('berilgan qiymatni kiriting');
write('x='); readln(x1);
writeln('jadvaldagi x ning qiymatlari');
for i:=0 to n do begin write('x[',i:1,']=' ) ; read(x[i]) end;
writeln('jadvaldagi y ning qiymatlari');
for i:=0 to n do begin write('y[',i:1,']= '); read(y[i]) end;
writeln('berilgan qiymat uchun y ning qiymati');
lagran(x1 ,n,x,y,y1);
writeln('y=',y1:5:6);
end.
```

Dastur natijasi

```
CRT - программа завершена
berilgan qiymatni kiriting
x=10
jadvaldagi x ning qiymatlari
x[0]=0
x[1]=1
x[2]=2
x[3]=3
jadvaldagi y ning qiymatlari
y[0]= 1
y[1]= -5
y[2]= 4
y[3]= 2
berilgan qiymat uchun y ning qiymati
y=-2504.000000
```

Lagranj ko’phadi uchun tuzilgan algoritm blok-sxemasi



Mustaqil bajarish uchun misollar.
Lagranj interpoliyatsion formulasi doir misollar.

X	$Y=e^x$	$Y=tgx$	$Y=\sin x$	$Y=\cos x$	$Y=\sqrt{x}$	X0
X_0=0.41	Y_0=1.5068	Y_0=0.4346	Y_0=0.0998	Y_0=0.9171	Y_0=0.6403	0.38
X_1=0.46	Y_1=1.5841	Y_1=0.4954	Y_1=0.4439	Y_1=0.8961	Y_1=0.6782	0.43
X_2=0.52	Y_2=1.6820	Y_2=0.5725	Y_2=0.4969	Y_2=0.8678	Y_2=0.7211	0.48
X_3=0.60	Y_3=1.8221	Y_3=0.6841	Y_3=0.5646	Y_3=0.8253	Y_3=0.7746	0.74
X_4=0.65	Y_4=1.9155	Y_4=0.7602	Y_4=0.6052	Y_4=0.7961	Y_4=0.8062	
X_5=0.72	Y_5=2.05444	Y_5=0.9316	Y_5=0.6593	Y_5=0.7518	Y_5=0.8485	
X_0=0,11	Y_0=1,1163	Y_0=0,1104	Y_0=0,1098	Y_0=0,9940	Y_0=0,3317	0,08
X_1=0,16	Y_1=1,1735	Y_1=0,1514	Y_1=0,1593	Y_1=0,9872	Y_1=0,4000	0,18
X_2=0,22	Y_2=1,2461	Y_2=0,2236	Y_2=0,2182	Y_2=0,9759	Y_2=0,4690	0,33
X_3=0,30	Y_3=1,3498	Y_3=0,3093	Y_3=0,2956	Y_3=0,9553	Y_3=0,5477	0,44
X_4=0,35	Y_4=1,4191	Y_4=0,3650	Y_4=0,3429	Y_4=0,9394	Y_4=0,5916	
X_5=0,42	Y_5=1,5220	Y_5=0,4466	Y_5=0,4078	Y_5=0,9131	Y_5=0,6481	
X_0=0.21	Y_0=1.2337	Y_0=0.2131	Y_0=0.2085	Y_0=0.9780	Y_0=0.4582	0.19
X_1=0.26	Y_1=1.2969	Y_1=0.2660	Y_1=0.2571	Y_1=0.9664	Y_1=0.5099	0.28
X_2=0.32	Y_2=1.3771	Y_2=0.3314	Y_2=0.3146	Y_2=0.9492	Y_2=0.5657	0.43
X_3=0.40	Y_3=1.4918	Y_3=0.4228	Y_3=0.3894	Y_3=0.9211	Y_3=0.6324	0.54
X_4=0.45	Y_4=1.5683	Y_4=0.4830	Y_4=0.4350	Y_4=0.9004	Y_4=0.6708	
X_5=0.52	Y_5=1.6820	Y_5=0.5726	Y_5=0.4969	Y_5=0.8678	Y_5=0.7211	
X_0=0.31	Y_0=1.3634	Y_0=0.3208	Y_0=0.3051	Y_0=0.9523	Y_0=0.5568	0.28
X_1=0.36	Y_1=1.4333	Y_1=0.3776	Y_1=0.3523	Y_1=0.9359	Y_1=0.6000	0.33
X_2=0.42	Y_2=1.5220	Y_2=0.4466	Y_2=0.4078	Y_2=0.9131	Y_2=0.6481	0.53
X_3=0.50	Y_3=1.6487	Y_3=0.5463	Y_3=0.4794	Y_3=0.8776	Y_3=0.7071	0.64
X_4=0.68	Y_4=1.7332	Y_4=0.6131	Y_4=0.5227	Y_4=0.8525	Y_4=0.7416	
X_5=0.62	Y_5=1.8539	Y_5=0.7139	Y_5=0.5810	Y_5=0.8139	Y_5=0.7874	
X_0=051	Y_0=1.6653	Y_0=0.5593	Y_0=0.4882	Y_0=08722	Y_0=0.7141	0.48
X_1=0.56	Y_1=1.7506	Y_1=0.6269	Y_1=0.5312	Y_1=0.8472	Y_1=0.7483	0.58
X_2=0.62	Y_2=1.8589	Y_2=0.7139	Y_2=0.5810	Y_2=0.8139	Y_2=0.7874	0.73
X_3=0.70	Y_3=2.0138	Y_3=0.8423	Y_3=0.6442	Y_3=0.7648	Y_3=0.8367	0.84
X_4=0.75	Y_4=2.1170	Y_4=0.9316	Y_4=0.6816	Y_4=0.7317	Y_4=0.8660	
X_5=0.82	Y_5=2.2705	Y_5=1.0717	Y_5=0.7311	Y_5=0.6822	Y_5=0.9055	

Nazorat savollari

12-Laboratoriya ishi

Nyuton interpolyatsion formulalari va ularni hisoblash algoritmi

Ishdan maqsad: Nyuton interpolyatsion formulalari va ularni hisoblash algoritmini tuzish.

Asosiy qism

N'yutonning 1-interpolyatsion formulasi. N'yutonning 1- interpolyatsion formulasi quyidagidan iborat bo'ladi:

$$P_n(x)=P_n(x_0+qh)=y_0+q\Delta y_0+\frac{q(q-1)}{2!}\Delta^2 y_0+...+\frac{q(q-1)\cdot ...\cdot(q-n+1)}{n!}\Delta^n y$$

N'yutonning 1-interpolyatsion formulasini [*a*, *b*] ning boshlangich nuqtalarida qo'llash qulay.

Agar $p=1$ bo'lsa, u holda$P_1(x)=y_0+q\Delta y_0$ko'rinishdagi chiziqli interpolyatsion formulaga, $p=2$bo'lganda esa

$$P_2(x)=y_0+q\Delta y_0+\frac{q(q-1)}{2}\Delta^2 y_0$$

ko'rinishdagi parabolik interpolyatsion formulaga ega bo'lamiz.

Misol. $u=lgx$ funksiyaning 2-jadvalda berilgan qiymatlaridan foydalanib, uning x=1001 bo'lgan holdagi qiymatini toping.

2-jadval

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1000	3,0000000	43214	- 426	8
1010	3,0043214	42788	- 418	9
1020	3,0086002	42370	- 409	8
1030	3,0128372	41961	- 401	
1040	3,0170333	41560		
1050	3,0211893			

Yechish. Chekli ayirmalar jadvalini tuzamiz.3- jadvaldan ko'rinib turibdiki, 3-tartibli chekli ayirma o'zgarmas, shu sababli (10.9) formula uchun n=3 olish yetarli:

$$y(x)=P_3(x)=y_0+q\Delta y_0+\frac{q(q-1)}{2!}\Delta^2 y_0+\frac{q(q-1)(q-2)}{3!}\Delta^3 y_0$$

$x=1001$ uchun $q=0,1$ ($h=10$). Shuning uchun

$$\lg 1001 = 3{,}0000000 + 0{,}1 \cdot 0{,}0043214 + \frac{0{,}1 \cdot 0{,}9}{2} \times$$
$$\times 0{,}0000426 + \frac{0{,}1 \cdot 0{,}9 \cdot 1{,}9}{6} \cdot 0{,}0000008 = 3{,}0004341$$

Misol. $y = f(x)$ funksiyaning 3-jadvalda berilgan qiymatlaridan foydalanib, uning x=10 bo'lgan holdagi qiymatini toping.

3-jadval

№	***x***	***y***
0	**0**	**1**
1	**1**	**-5**
2	**2**	**4**
3	**3**	**2**

Yechish: Lagranj formulasidan foydalanib ABC Pascal dasturida natija olamiz. Quyida dastur matni va natija keltirildi.

Haqiqatan ham, izlanayotgan ko'phad 3-darajali ko'phad bo'ladi, ya'ni $L(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$. Bu yerda n=3 ekanligini ko'rishimiz mumkin. $L(x)$ ko'phadni aniqlaymiz.

$$L(x) = y_0 \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} + y_1 \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} +$$
$$+ y_2 \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} + y_3 \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$

yoki $L(x) \approx -4{,}3333x^3 + 20{,}500x^2 - 22{,}167x + 1$ ko'phadga ega bo'lamiz va x=10 uchun hisoblaymiz:

$$L(10) \approx -4333 + 2050 - 222 + 1 = -2504$$

Nyuton interpoliyatsion formulasi doir misollar.

$y = f(x)$ funksiyaning jadvalda berilgan qiymatlaridan foydalanib, uning x=32 bo'lgan holdagi qiymatini toping.

X	10	15	20	25	30	35	40
Y	1,24	3,71	-6,45	11,08	12,24	-1,57	-12,88

Nazorat savollari

13-14-Laboratoriya ishi

Empirik formulalar. Funksiyalarni eng kichik kvadratlar usuli bilan approksimatsiyalash va uning algoritmi

Ishning maqsadi: Tajriba natijalarini eng kichik kvadratlar usuli bilan approksimatsiyalash bo'yicha amliy ko'nikmani shakllantirish

Nazariy qism

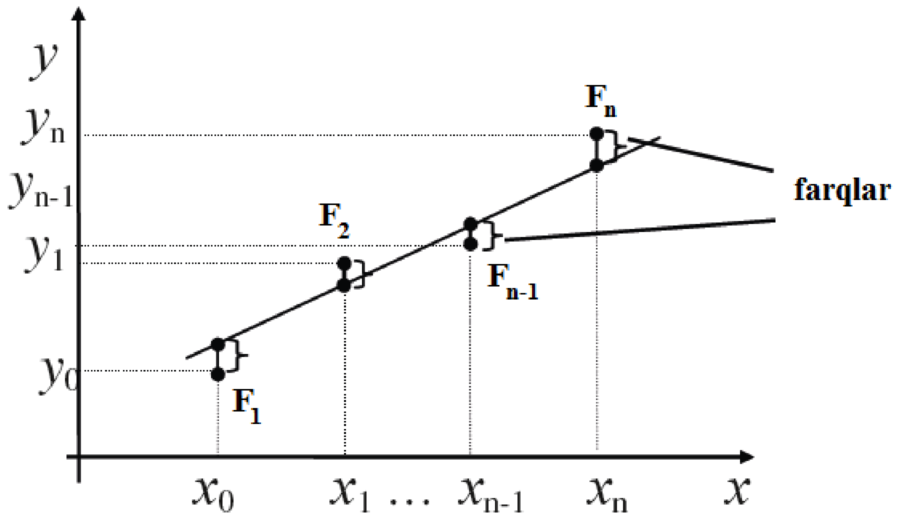
Tajriba natijalarini q'ayta ishlash. Eslatib o'tamiz, interpolyatsiya deganda erkli o'zgaruvchining diskret nuqtalari bilan funksiyaning shu nuqtalardagi mos q'iymatlari orasidagi munosabati ma'lum bo'lgan holda funksional bog'lanishning taqribiy yoki aniq analitik ifodasini tuzish tushuniladi. Ko'pincha kuzatishlar va tajribalar orqali empirik formulalarni keltirib chiq'arish mumkin. Masalan, haroratning ko'tarilishi yoki aksincha pasayishini, simob ustunining ko'tarilishi yoki pasayishiga qarab bilish mumkin. Demak, harorat bilan simob ustini o'rtasidagi chiziqli bog'lanish borligini tajriba orqali bilish mumkin. Bunday masalalarni yechishda **eng kichik kvadratlar** usulidan foydalanamiz.

Emperik bog'liqlikni aniqlash. Qandaydir funksiyaning qiymatlari jadval ko'rinishida berilgan bo'lsin. U holda bu funksiyani jadval funksiya ham deb ataymiz.

X	X_1	X_2	...	X_n
Y	Y_1	Y_2	...	Y_n

Bu yrda berilgan tajriba natijalarini bog'lovchi emperik funksiya sifatida ushbu $y = f(x)$ funksiyani aniqlash masalasini ko'rib chiqamiz.

Jadvaldagi qiymatlar bo'yicha $F(x_i, y_i)$ nuqtalarni Dekart koordinatalar sistemasida tasvirlaymiz.



$y = f(x)$ funksiya uchun $y_i \approx f(x_i)$ shart o'rinli bo'lsin. Bu yerda shart xatoligi $\varepsilon_i = y_i^0 - y_i$ qabul qilinsin. Bu yerda $y_i^0 = f(x_i)$.

$\varphi_0(x), \varphi_1(x), \varphi_2(x), ..., \varphi_m(x)$ - bazis funksiyalar bo'lsin va bu funksiyalar yordamida $y = \Phi_m(x) \equiv c_0\varphi_0(x) + c_1\varphi_1(x) + ... + c_m\varphi_m(x)$ (13.1) funksiyani hosil qilamiz va bu yerda $\Phi_m(x)$ ko'phadga ega bo'lamiz.

(13.1) – formulada $c_j \quad (j=1,...,m)$ koeffitsiyentlarni aniqlashda eng kichik kvadratlar usulidan foydalanamiz ya'ni,

$$\delta_m = \sum_{i=0}^{n}\left(y_i - \Phi_m(x_i)\right)^2 = \sum_{i=0}^{n}\left(y_i - c_0\varphi_0(x_i) - ... - c_m\varphi_m(x_i)\right)^2 \quad (13.2)$$

funksiyaning minimumini topamiz. Demak, shunday $c_j \quad (j=1,...,m)$ noma'lumlarni aniqlash lozimki natijada δ_m funksiyaning qiymati eng kichik bo'lsin. Ma'lumki, ihtiyoriy x uchun $\delta_m \geq 0$ shart bajariladi. δ_m funksiyaning $c_j \quad (j=1,...,m)$ argumentlaridagi birinchi tartibli hususiy hosilalarini hisoblaymiz va ularni nolga tenglaymiz. Natijada quyidagi tenglamalar sistemasiga ega bo'lamiz.

$$\begin{cases} \dfrac{\partial\delta_m}{\partial c_0} = -2\sum\limits_{i=0}^{n}(y_i - c_0\varphi_0(x_i) - ... - c_m\varphi_m(x_i))\cdot\varphi_0(x_i) = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{\partial\delta_m}{\partial c_m} = -2\sum\limits_{i=0}^{n}(y_i - c_0\varphi_0(x_i) - ... - c_m\varphi_m(x_i))\cdot\varphi_m(x_i) = 0 \end{cases} \quad (13.3)$$

Agar $(f,g) = \sum\limits_{i=0}^{n} f(x_i)\cdot g(x_i)$ tenglikdan foydalansak uholda (13.3) –sistemani quyidagicha yozish mumkin bo'ladi.

$$\begin{cases} c_0(\varphi_0,\varphi_0) + c_1(\varphi_0,\varphi_1) + ... + c_m(\varphi_0,\varphi_m) = (\varphi_0, y) \\ c_0(\varphi_1,\varphi_0) + c_1(\varphi_1,\varphi_1) + ... + c_m(\varphi_1,\varphi_m) = (\varphi_1, y) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ c_0(\varphi_m,\varphi_0) + c_1(\varphi_m,\varphi_1) + ... + c_m(\varphi_m,\varphi_m) = (\varphi_m, y) \end{cases} \quad (13.4)$$

$\varphi_k(x) = x^k$ ekanligidan (13.4) sistemani quyidagicha yozish mumkin.

$$\sum_{j=0}^{m}\left(\sum_{j=0}^{n} x_i^{j+k}\right)c_j = \sum_{i=0}^{n} y_i x_i^k \,, \quad (k = 0,1,...,m) \qquad (13.5)$$

Endi m=1 va m=2 uchun (7.5) sistemani aniqlaymiz.

m=1 bo'lsin. U holda $P_1(x) = c_0 + c_1x$ chiziqli ko'phadga ega bo'lamiz va c_0 , c_1 larni aniqlash uchun (7.5) sistemaning ko'rinishi quyidagicha bo'ladi.

$$\begin{cases} (n+1)c_0 + (\sum\limits_{i=0}^{n} x_i)c_1 = \sum\limits_{i=0}^{n} y_i \\ (\sum\limits_{i=0}^{n} x_i)c_0 + (\sum\limits_{i=0}^{n} x_i^2)c_1 = \sum\limits_{i=0}^{n} y_i x_i. \end{cases}$$

m=2 bo'lsin. U holda $P_1(x) = c_0 + c_1x + c_2x^2$ kvadrat uchhadga ega bo'lamiz va c_0 , c_1 ,c_2 larni aniqlash uchun (13.5) sistemaning ko'rinishi quyidagicha bo'ladi.

$$\begin{cases} (n+1)c_0 + (\sum_{i=0}^{n} x_i)c_1 + (\sum_{i=0}^{n} x_i^2)c_2 = \sum_{i=0}^{n} y_i \\ (\sum_{i=0}^{n} x_i)c_0 + (\sum_{i=0}^{n} x_i^2)c_1 + (\sum_{i=0}^{n} x_i^3)c_2 = \sum_{i=0}^{n} y_i x_i \\ (\sum_{i=0}^{n} x_i^2)c_0 + (\sum_{i=0}^{n} x_i^3)c_1 + (\sum_{i=0}^{n} x_i^4)c_2 = \sum_{i=0}^{n} y_i x_i^2 \end{cases}$$

Misol-1. Berilgan jadval qiymatlari asosida tajriba natijalarini eng kichik kvadratlar usuli bilan approksimatsiyalang (T=4,2).

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	5	4	8	1	11	9	2	0	7	4

Yechish. m=1 bo'lsin. U holda quyidagi jadvalni MS Excel dasturi yordamida hisoblaymiz.

N	X	Y	XY	X2
1	0	5	0	0
2	1	4	4	1
3	1,5	8	12	2,25
4	3	1	3	9
5	4	11	44	16
6	5	9	45	25
7	8	2	16	64
8	8,5	0	0	72,25
9	9	7	63	81
10	9,5	4	38	90,25
	49,5	51	225	360,75

$P_1(x) = c_0 + c_1 x$ regressiyani noma'lum koeffitsentlarini hisoblaymiz. Buning uchun tenglamalar sitemasini tuzamiz (n=9).

$$\begin{cases} 10c_0 + 49{,}5c_1 = 51 \\ 49{,}5c_0 + 360{,}75c_1 = 225 \end{cases}$$

Bu sistemani yechib $c_0 = 6{,}274$, $c_1 = -0{,}237$ ildizlarga ega bo'lamiz. U holda $P_1(x)$ regressiya ko'phadining ko'rinishi quyidagicha bo'ladi: $P_1(x) = 6{,}274 - 0{,}27x$

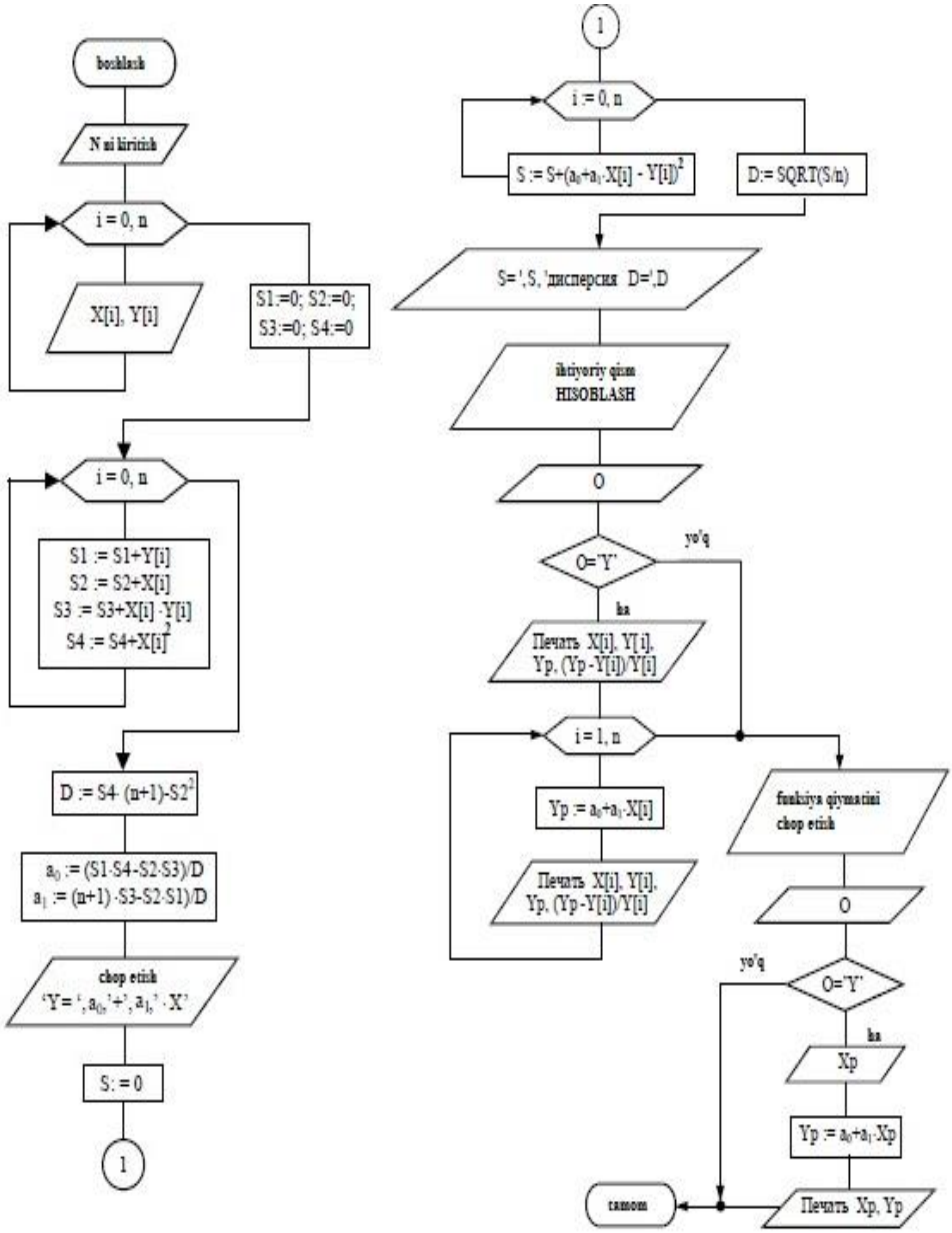
Ushbu jadvalga ega bo'lamiz:

Y	P(X)
5	6,2741413
4	6,036941
8	5,9183409
1	5,5625405

11	5,3253402
9	5,08814
2	4,3765392
0	4,2579391
7	4,139339
4	4,0207388
51	51

$y(4,2) = 6,274 - 0,27 \cdot 4,2 = 5,278$

Eng kichik kvadratlar usuliga tuzilgan algoritm blok-sxemasi



1-misolda keltirilgan tajriba natijalarini qayta ishlash uchun eng kichik kvadratlar usuliga tuzilgan ABC Pascal dasturlash tilidagi dastur matni keltirilgan. Bu yerda $T = x_0 = 4{,}2$ deb qaralsin.

```
Program Kvadrat; uses crt;
const n=10;
var x0,y0,a,b,c1,c2,p1,p2:real;
i:integer;
x,y:array[1..n] of real;
begin
 writeln('Qaysi qiymat uchun hisoblaymiz');
write('x0='); readln(x0);
writeln('Massiv elementlarini kiriting');
for i:=1 to n do begin
write('x',I:1,'=');readln(x[I]);
write('y',I:1,'=');readln(y[I]); end;
c1:=0; c2:=0; p1:=0; p2:=0;
for I:=1 to n do begin
c1:=c1+x[I]*x[I];
c2:=c2+x[I];
p1:=p1+x[I]*y[I];
p2:=p2+y[I];
 end;
a:=(p1*n-p2*c2)/(c1*n-c2*c2);
b:=(p2-a*c2)/n;
writeln('a=',a);writeln('b=',b);
y0:=a*x0+b;
writeln('y0=',y0:5:6);
begin
   for I:=1 to n do begin
          y0:=a*x[i]+b;
          writeln('y',I:1,'=',y0:5:6);end;end;
end.
```

Dastur natijasi

```
CRT - программа завершена
y5=11
x6=5
y6=9
x7=8
y7=2
x8=8.5
y8=0
x9=9
y9=7
x10=9.5
y10=4
a=-0.237200259235256
b=6.27414128321452
y0=5.277900
y1=6.274141
y2=6.036941
y3=5.918341
y4=5.562541
y5=5.325340
y6=5.088140
y7=4.376539
y8=4.257939
y9=4.139339
y10=4.020739
_
```

Mustaqil yechish uchun misollar

Berilgan jadval qiymatlari asosida tajriba natijalarini eng kichik kvadratlar usuli bilan approksimatsiyalang (chiziqli regressiya T=5,3).

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	1	1,12	1,87	2,25	5,01	5,27	7,14	9,21	8,42	10,9

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	2	3	4	5	6,5	7,5	8	8,5
Y	-0,4	1,8	3,7	2,8	4,1	4,8	7,2	8,01	7,57	9,21

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	1	0,7	2	2,4	4,5	4,8	7,77	9	8,8	10

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0	1,5	2	2,5	4,2	5,5	8,5	7,8	8,9	11,5

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0.5	1,5	2,5	2,9	3,7	5,1	7,8	7	8	10,2

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0,9	1,1	1,5	2,9	3	4,1	8,5	8,9	8	9,4

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0,8	1,2	1,5	2,2	3,8	4,9	8,8	9,9	8,5	9

****

N	1	2	3	4	5	6	7	8	9	10
X	0	1	1,5	3	4	5	8	8,5	9	9,5
Y	0,1	1,9	1,6	2,2	4,5	4,5	8,8	9,4	10	8,7

Nazorat savollari

15-Laboratoriya ishi

Transport masalasini yechishning eng kichik harajatlar usuli va uning algoritmi

Ishdan maqsad: Transport masalasini yechishning eng kichik harajatlar usuli va uning algoritmini topish.

Nazariy qism

Transport masalasi qo'llanilish sohasi keng bo'lgan tushunchalar sirasiga kiritiladi. Oddiy holda uning berilshini quyidagicha keltirish mumkin.

$A_1, A_2, ..., A_n$ jo'natish punktlarida mos ravishda $a_1, a_2, ..., a_n$ zahira miqdori (yuklar) mavjud. $B_1, B_2, ..., B_k$ qabul qilish punktlari uchun mos ravishda $b_1, b_2, ..., b_k$ miqdordagi yukga ihtiyoj mavjud. Har bir A_i dan har bir B_j ga bir birlik yukni tashish uchun ketadigan sarf harajat miqdori $c_{ij}\ (i = 1, ..., n;\ j = 1, ...k)$ ma'lum bo'lsa, u holda yuk tashishning shunday optimal variantini tanlash kerakki, natijada umumiy sarf harajat miqdori eng kichik bo'lsin.

Bu yerda zahira va ehtiyoj miqdorlari teng bo'lgan hol (yopiq masala) uchin masala berilgan deb hisoblaylik, ya'ni $\sum_i a_i = \sum_j b_j$.

Transport masalasini yechishning matematik modelini quramiz.

Aytaylik, A_i dan B_j ga jo'natiladigan yuk miqdori x_{ij} ga teng bo'lsin. U holda umumiy sarf harajat miqdori $\sum_{ij} c_{ij} x_{ij}$ ga teng bo'ladi. Shuningdek, ayrim cheklanishlarga egamiz, yani unga ko'ra quyidagi sistemani hosil qilamiz:

$$\begin{cases} x_{11} + x_{12} + ... + x_{1k} = a_1 \\ x_{21} + x_{22} + ... + x_{2k} = a_2 \\ \dots\dots\ \dots\dots\dots\dots \\ x_{n1} + x_{n2} + ... + x_{nk} = a_n \\ x_{11} + x_{21} + ... + x_{n1} = b_1 \\ \dots\dots\ \dots\dots\dots\dots \\ x_{1n} + x_{2n} + ... + x_{nk} = b_k \end{cases} \qquad (15.1)$$

Masala shartiga ko'ra (15.1) cheklanishlar sistemasining shunday yechimini topish talab etiladiki, natijada quyidagi munosabat o'rinli bo'lsin:

$$F = \sum_{i,j}^{n,k} c_{ij} x_{ij} \to \min \qquad (15.2)$$

Transport masalasiga oid misollar keltiramiz va ularni hisoblash algoritmi bilan tanishamiz.

1 – masala. Zahira nuqtalari A_1, A_2 va A_3 da mos ravishda 300 birlik, 400 birlik va 500 birlik yuk mavjud. Qabul qilish nuqtalari B_1, B_2, B_3 va B_4 lar-ning mos ravishda 280 birlik, 300 birlik, 320 birlik va 300 birlik yukga ehtiyoji bor. Agar sarf harajat matrisasi quyidagi ko'rinishda bo'lsa, ya'ni

$$C = \begin{pmatrix} 5 & 9 & 7 & 10 \\ 8 & 12 & 9 & 13 \\ 7 & 13 & 10 & 18 \end{pmatrix}$$

u holda yuk tashishning shunday optimal variantini tuzing, natijada sarf harajat eng kam bo'lsin.

Yechish. Berilgan masala transport masalai bo'lib, uni ychishda dastlab, sarf harajat matrisasi elementi ma'nosini aniqlashtirib olamiz. Xususan, C matrisaning ikkinchi satri, to'rtinchi ustunida joylashgan 13 soni A_2 yuk jo'natish nuqtasidan B_4 yuk qabul qilish nuqtasiga 1 birlik yukni tashish uchun zarur bo'lgan harajat kattaligi. Yuqorida keltirilganidek, berilgan masala transport (yopiq) masalasi bo'lib, uning cheklanishlar sistemasi va maqsad funksiyasi x_{ij} ($i = 1,...,3;\ j = 1,...,4$) larga nisbatan quyidagi ko'rinishda bo'ladi. Bu yerda x_{ij} - A_i dan B_j ga tashiladigan yuk miqdori.

$$\begin{cases} x_{11} + x_{12} + x_{13} + x_{14} = 300 \\ x_{21} + x_{22} + x_{23} + x_{24} = 400 \\ x_{31} + x_{32} + x_{33} + x_{34} = 500 \\ x_{11} + x_{21} + x_{31} = 280 \\ x_{12} + x_{22} + x_{32} = 300 \\ x_{13} + x_{23} + x_{33} = 320 \\ x_{14} + x_{24} + x_{34} = 300 \end{cases} \qquad (15.3)$$

$$F = 5x_{11} + 9x_{12} + 7x_{13} + 10x_{14} + 8x_{21} + 12x_{22} + 9x_{23} + 13x_{24} + 7x_{31} +$$
$$+ 13x_{32} + 10x_{33} + 18x_{34}$$

Transport masalasini hisoblashda yuk tarqatish jadvali tuziladi va yuk taqsimoti "eng kam narx" usuli yordamida amalga oshiriladi. Buni berilgan masala bo'yicha ko'rib chiqaylik.

1-variant. Quyidagi jadvalni qaraylik (1-jadval).

1-jadval.

	B1	B2	B3	B4	Zahira
A1	5	9	7	10	300
A2	8	12	9	13	400
A3	7	13	10	18	500
Ehtiyoj	280	300	320	300	1200

Bu jadvalni "eng kam narx" usuli tamoyiliga ko'ra to'ldiramiz (2-jadval).

2-jadval

	B1	B2	B3	B4	Zahira
A1	5 / **100**	9 / **100**	7 / **100**	10	300
A2	8 / **100**	12 / **100**	9 / **100**	13 / **100**	400
A3	7 / **80**	13 / **100**	10 / **120**	18 / **200**	500
Ehtiyoj	280	300	320	300	1200

Ohirgi jadval bo'yicha F - maqsad funksiya qiymatini hisoblaymiz:

$$F = 500 + 900 + 700 + 800 + 1200 + 900 + 1300 + 560 + 1300 +$$
$$+ 1200 + 3600 = 12960$$

1 – variant tanlanmasiga ko'ra maqsad funksiya qiymati 12960 ga teng ekan. Endi ikkinchi variant tanlanmalarini quyidagicha keltiramiz (3-jadval):

3-jadval

	B1	B2	B3	B4	Zahira
A1	5 / **200**	9 / **50**	7 / **50**	10	300
A2	8 / **80**	12 / **150**	9 / **100**	13 / **70**	400
A3	7	13 / **100**	10 / **170**	18 / **230**	500
Ehtiyoj	280	300	320	300	1200

Ohirgi jadval bo'yicha F - maqsad funksiya qiymatini hisoblasak, u holda 13190 qiymatga ega bo'lamiz. Demak 2 – variant optimal emasligi aniq.

1 – variantning optimallikga tekshirish uchun odatda potensiallar usulidan foydalaniladi. Lekin biz amaliy dastur yordamida berilgan masalaning optimal yechimini topishga harakat qilamiz. Buning uchun MS Excel dasuridan foydalanamiz. Uning «ПОИСК РЕШЕНИЯ» ustqurmasi yordamida optimallash masalalarini osonlik bilan hisoblash mumkin. Demak, «ПОИСК РЕШЕНИЯ» ustqurmasidan foydalangan holda transport masalasini yechish algoritmini keltiramiz. Bu quyidagi ketma – ketlikda amalga oshiriladi.

1. Berilgan ma’lumotlarni A1:F8 diapazonga kiritamiz.

2. B3:E3, B5:E5 va B7:E7 diapazonlarga nol yozib chiqamiz

3. G3 katakga =СУММ(B3:E3) formulani yozamiz va uni G5 va G7 kataklarga nusxalaymiz.

4. B9 katakga =B3+B5+B7 formulani yozamiz va uni C9, D9 va E9 kataklarga nusxalaymiz.

5. H2 katakga =СУММПРОИЗВ(B2:E2;B3:E3) formulasin yozamiz va uni H4 va H6 kataklarga nusxalaymiz.

6. H8 katakga =H2+H4+H6 formulani yozib qo’yamiz va «ПОИСК РЕШЕНИЯ» ustqurmasini ishga tushirishga tayyorgarlikni yakunlaymiz (6.9-rasm).

7. Kursorni H8 katakda qoldirib, «ПОИСК РЕШЕНИЯ» ustqurmasini ishga tushiramiz va kerakli to’ldirishlarni amalga oshiramiz (6.10-rasm).

8. «ПОИСК РЕШЕНИЯ» muloqat oynasidagi «ПАРАМЕТРЫ» tugmasini bosish bilan uning muloqat oynasiga o’tamiz va u yerda «НЕОТРИЦАТЕЛЬНЫЕ ЗНАЧЕНИЯ» ga bayroqcha o’rnatamiz (6.11-rasm).

9. OK tugmasini bosib yana «ПОИСК РЕШЕНИЯ» muloqat oynasiga qaytamiz va «ВЫПОЛНИТЬ» tugmasini bosib ishni yakunlaymiz.

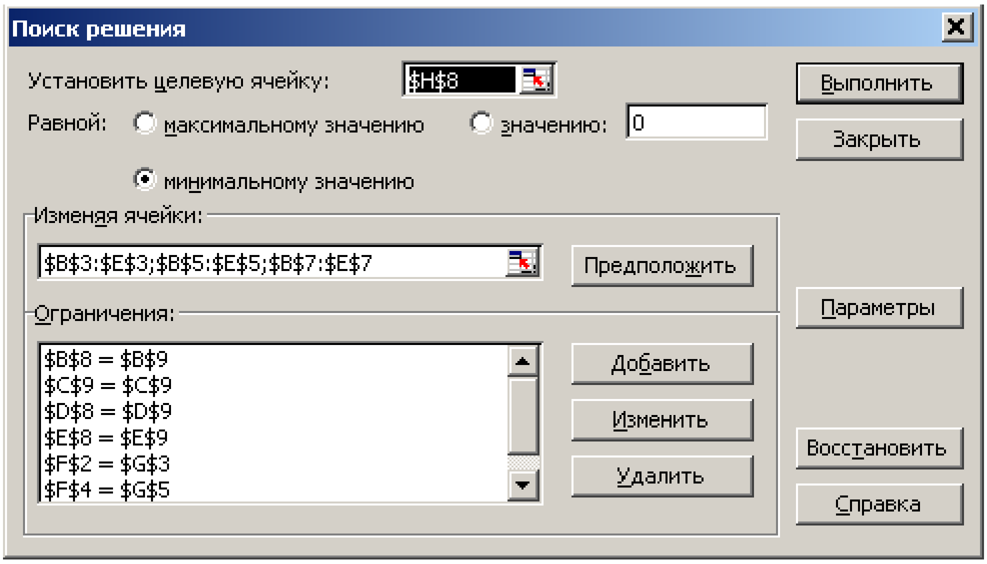
Natijada berilgan masala uchun optimal yechimni aniqlaymiz (6.12-rasm). Unga ko’ra maqsad funksiyaning eng kichik qiymati 11660 ga tengligi kelib chiqadi. Bu izlanayotgan optimal yechim.

Bu yerda shuni ta’kidlash lozimki, yuqoridagi algoritmning 7 – qadamida umumiylik uchun aytib o’tilgan jumlada zukko o’quvchi «ПОИСК РЕШЕНИЯ» muloqat oynasidagi kerakli ishlarni o’zi bajara olishiga ishonamiz, chunki bu alohida izohlab o’tiladigan darajada murakkab jarayon emas.

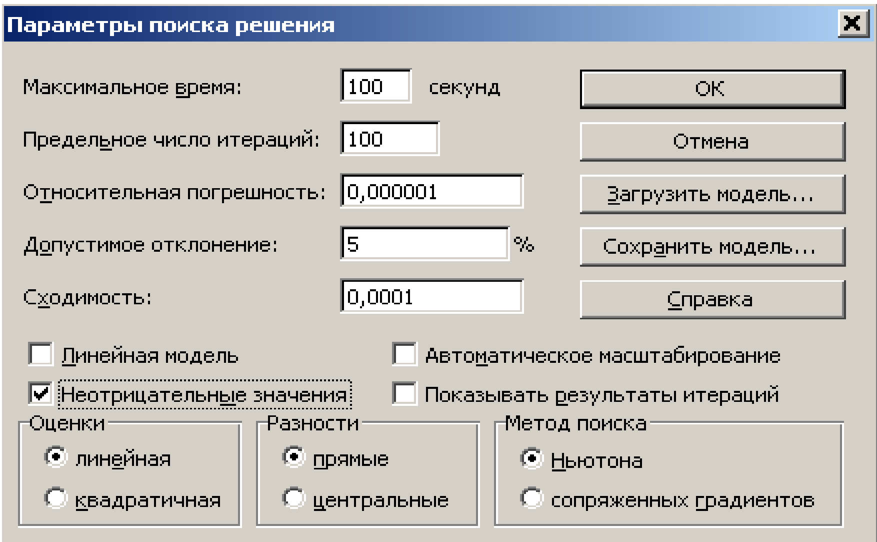
Keyingi masalalarni yechishda yuqorida keltirilgan algoritm bo’yicha to’g’ridan – to’g’ri hisoblashlarni amalga oshiramiz, ya’ni MS Excel dasturida bajarilgan ishlar ketma – ketligini izohlarsiz keltiramiz.

	A	B	C	D	E	F	G	H
1		**B1**	**B2**	**B3**	**B4**	**Zahira**		
2	**A1**	5	9	7	10	**300**		**0**
3		**0**	**0**	**0**	**0**		**0**	
4	**A2**	8	12	9	13	**400**		**0**
5		**0**	**0**	**0**	**0**		**0**	
6	**A3**	7	13	10	18	**500**		**0**
7		**0**	**0**	**0**	**0**		**0**	
8	**Ehtiyoj**	**280**	**300**	**320**	**300**	**1200**	**F= 0**	
9		**0**	**0**	**0**	**0**			

15.1-rasm. «ПОИСК РЕШЕНИЯ» ga o'tish uchun boshlang'ich loyiha.



15.2-rasm. To'ldirilgan «ПОИСК РЕШЕНИЯ» muloqat oynasi.



15.3-rasm. «Параметры поиска решиния» mulaqot oynasi ko'rinishi

	A	B	C	D	E	F	G	H
1		B1	B2	B3	B4	Zahira		
2	A1	5	9	7	10	300		2886,686
3		0	113,3	0	186,7		300	
4	A2	8	12	9	13	400		4421,948
5		0	122,9	163,8	113,3		400	
6	A3	7	13	10	18	500		4351,367
7		280	63,79	156,2	0		500	
8	Ehtiyoj	280	300	320	300	1200	F= 11660	
9		280	300	320	300			

15.4-rasm. Transport masalasini MS Excelda hisoblash natijasi

2–masala. Ishlab chiqarish korxonasida ta'mirlash ishlari olib borilmoqda. Ikki turdagi bir xil vazifani bajaruvchi nosoz qurilmalarni ta'mirlash rejalashtirilgan. Birinchi turdan 12 ta va ikkinchi turdan 18 ta qurilma ta'mir talab holatda. Ularni ta'mirlashda har biri uchun bir xil besh turdagi ehtiyoj qismlar ishlatiladi va ularning texnik ma'lumotlari quyidagi jadvalda keltirilgan (4-jadval):

4-jadval.

	1-tur e.q.	2-tur e.q.	3-tur e.q.	4-tur e.q.	5-tur e.q.	**Ehtiyoj**
1-tur qurilma	9	7	7	4	7	12 ta
2-tur qurilma	6	5	8	7	3	18 ta
Zahira	164 ta	158 ta	200 ta	150 ta	120 ta	

Zahira qismlar soni barcha nosoz qurilmalarni ta'mirlash uchun yetarli emas. Shuning uchun ta'mirlanadigan qurilmalar sonini ajratish zaruriyati kelib chiqadi. Agar birinchi va ikkinchi tur qurilamlarning foydali ish koeffisyentlari mos ravishda $k_1 = 0,75$ va $k_2 = 0,5$ bo'lsa, ta'mirlashning shunday optimal rejasini tuzingki, natijada ishlab chiqarish samaradorligi eng yuqori bo'lsin.

Yechish. 1 – qurilmadan x dona, 2 – qurilmadan y dona ta'mirlansin deylik. U holda ishlab chiqarish samaradorligini ifodalovchi magsad funksiyasi uchun quyidagi formulani yozishimiz mumkin: $F(x, y) = 0,75x + 0,5y \rightarrow \max$.

Shuningdek, keltirilgan jarayon uchun zahira cheklanishlari sistemasini yozamiz:

$$\begin{cases} 9x + 6y \le 164 \\ 7x + 5y \le 158 \\ 7x + 8y \le 200 \\ 4x + 7y \le 150 \\ 7x + 3y \le 120 \\ 0 \le x \le 12 \\ 0 \le y \le 18 \end{cases}$$

Oldingi masaladagi kabi «ПОИСК РЕШЕНИЯ» ustqurmasidan foydalanib, berilganlar asosida hisoblaymiz (15.6-rasm):

	A	B	C	D	E
1		x	y		
2		9	6	164	164
3		7	5	131	158
4		7	8	159	200
5		4	7	113	150
6		7	3	112	120
7		12	9	14	
8		x	y	F	
9		12	18		

15.6-rasm. Masala yechimini MS Excel dasturida hisoblash.

Hisoblash natijalariga ko'ra, zahira miqdorini hisobga olib, birinchi qurilmadan 12 dona, ikkinchi qurilmadan 9 dona ta'mirlansa, ishlab chiqarish samaradorligi eng yuqori bo'lar ekan.

Mustaqil yechish uchun misollar.

1. Transport masalasini yeching. Bu yerda A_1 va A_2 yuk jo'natish zahiralari, B_1, B_2 va B_3 lar yuk qabul qilish ehtiyoj kattaliklari, C - harajatlar matrisasi.

№	B1	B2	B3	A1	A2
1	200	300	250	350	400
2	150	250	200	300	300
3	100	200	300	400	200
4	500	400	300	250	950
5	400	250	250	400	500
6	250	350	250	400	450
7	200	300	400	350	550
8	350	450	400	700	500

9	350	400	550	600	700
10	250	350	550	650	500
11	240	360	420	520	500
12	340	260	450	500	550

$$C = \begin{pmatrix} 9 & 4 & 7 \\ 12 & 8 & 5 \end{pmatrix}$$

Nazorat savollari

1. Transport masalasi qanday masala va uning taqbiqlari?
2. Transport masalasining qanday turlari mavjud?
3. Transport masalasini Excel dasturida hisoblash qanday algoritmda bajariladi?

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